THE THEORETICAL MINIMUM

WHAT YOU NEED *to* KNOW *to* START DOING PHYSICS

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To our spouses—
those who have chosen to put up with us,
and to the students of Professor Susskind's
Continuing Education Courses

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Preface

I've always enjoyed explaining physics. For me it's much more than teaching: It's a way of thinking. Even when I'm at my desk doing research, there's a dialog going on in my head. Figuring out the best way to explain something is almost always the best way to understand it yourself.

About ten years ago someone asked me if I would teach a course for the public. As it happens, the Stanford area has a lot of people who once wanted to study physics, but life got in the way. They had had all kinds of careers but never forgot their one-time infatuation with the laws of the universe. Now, after a career or two, they wanted to get back into it, at least at a casual level.

Unfortunately there was not much opportunity for such folks to take courses. As a rule, Stanford and other universities don't allow outsiders into classes, and, for most of these grown-ups, going back to school as a full-time student is not a realistic option. That bothered me. There ought to be a way for people to develop their interest by interacting with active scientists, but there didn't seem to be one.

That's when I first found out about Stanford's Continuing Studies program. This program offers courses for people in the local nonacademic community. So I thought that it might just serve my purposes in finding someone to explain physics to, as well as their purposes, and it might also be fun to teach a course on modern physics. For one academic quarter anyhow.

It was fun. And it was very satisfying in a way that teaching undergraduate and graduate students was sometimes not. These students were there for only one reason: Not to get credit, not to get a degree, and not to be tested, but just to learn and indulge their curiosity. Also, having been "around the block" a few times, they were not at all afraid to ask questions, so the class had a lively vibrancy that academic classes often lack. I decided to do it again. And again.

What became clear after a couple of quarters is that the students were not completely satisfied with the layperson's courses I was teaching. They wanted more than the *Scientific American* experience. A lot of them had a bit of background, a bit of physics, a rusty but not dead knowledge of calculus, and some experience at solving technical problems. They were ready to try their hand at learning the real thing—with equations. The result was a sequence of courses intended to bring these students to the forefront of modern physics and cosmology.

Fortunately, someone (not I) had the bright idea to video-record the classes. They are out on the Internet, and it seems that they are tremendously popular: Stanford is not the only place with people hungry to learn physics. From all over the world I get thousands of e-mail messages. One of the main inquiries is whether I will ever convert the lectures into books? *The Theoretical Minimum* is the answer.

The term *theoretical minimum* was not my own invention. It originated with the great Russian physicist Lev Landau. The TM in Russia meant everything a student needed to know to work under Landau himself. Landau was a very demanding man: His theoretical minimum meant just about everything he knew, which of course no one else could possibly know.

I use the term differently. For me, the theoretical minimum means just what you need to know in order to proceed to the next level. It means not fat encyclopedic textbooks that explain everything, but thin books that explain everything important. The books closely follow the Internet courses that you will find on the Web.

Welcome, then, to *The Theoretical Miniumum*—Classical Mechanics, and good luck!

Leonard Susskind
Stanford, California, July 2012

I started to teach myself math and physics when I was eleven. That was forty years ago. A lot of things have happened since then—I am one of those individuals who got sidetracked by life. Still, I have learned a lot of math and physics. Despite the fact that people pay me to do research for them, I never pursued a degree.

For me, this book began with an e-mail. After watching the lectures that form the basis for the book, I wrote an e-mail to Leonard Susskind asking if he wanted to turn the lectures into a book. One thing led to another, and here we are.

We could not fit everything we wanted into this book, or it wouldn't be *The Theoretical Minimum*—Classical Mechanics, it would be A-Big-Fat-Mechanics-Book. That is what the Internet is for: Taking up large quantities of bandwidth to display stuff that doesn't fit elsewhere! You can find extra material at the website www.madscitech.org/tm. This material will include answers to the problems, demonstrations, and additional material that we couldn't put in the book.

I hope you enjoy reading this book as much as we enjoyed writing it.

George Hrabovsky
Madison, Wisconsin, July 2012

Lecture 1: The Nature of Classical Physics

Somewhere in Steinbeck country two tired men sit down at the side of the road. Lenny combs his beard with his fingers and says, "Tell me about the laws of physics, George." George looks down for a moment, then peers at Lenny over the tops of his glasses. "Okay, Lenny, but just the minimum."

What Is Classical Physics?

The term *classical physics* refers to physics before the advent of quantum mechanics. Classical physics includes Newton's equations for the motion of particles, the Maxwell-Faraday theory of electromagnetic fields, and Einstein's general theory of relativity. But it is more than just specific theories of specific phenomena; it is a set of principles and rules—an underlying logic—that governs all phenomena for which quantum uncertainty is not important. Those general rules are called *classical mechanics*.

The job of classical mechanics is to predict the future. The great eighteenth-century physicist Pierre-Simon Laplace laid it out in a famous quote:

> *We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those*

of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

In classical physics, if you know everything about a system at some instant of time, and you also know the equations that govern how the system changes, then you can predict the future. That's what we mean when we say that the classical laws of physics are *deterministic*. If we can say the same thing, but with the past and future reversed, then the same equations tell you everything about the past. Such a system is called *reversible*.

Simple Dynamical Systems and the Space of States

A collection of objects—particles, fields, waves, or whatever—is called a *system*. A system that is either the entire universe or is so isolated from everything else that it behaves as if nothing else exists is a *closed* system.

Exercise 1: Since the notion is so important to theoretical physics, think about what a closed system is and speculate on whether closed systems can actually exist. What assumptions are implicit in establishing a closed system? What is an open system?

To get an idea of what deterministic and reversible mean, we are going to begin with some extremely simple closed systems. They are much simpler than the things we usually study in physics, but they satisfy rules that are rudimentary versions of the laws of classical mechanics. We begin with an example that is so simple it is trivial. Imagine an abstract object that has only one state. We could think of it as a coin glued to the table—forever showing heads. In physics jargon, the collection of all states occupied by a system is its space of states, or, more simply, its *state-space*. The state-space is not ordinary space; it's a mathematical set whose elements label the possible states of the system. Here the state-space consists of a single point—namely Heads (or just H)—because the system has only one state. Predicting the future of this system is extremely simple: Nothing ever happens and the outcome of any observation is always H.

The next simplest system has a state-space consisting of two points; in this case we have one abstract object and two possible states. Imagine a coin that can be either Heads or Tails (H or T). See Figure 1.

Figure 1: The space of two states.

In classical mechanics we assume that systems evolve smoothly, without any jumps or interruptions. Such behavior is said to be *continuous*. Obviously you cannot move between Heads and Tails smoothly. Moving, in this case, necessarily occurs in discrete jumps. So let's assume that time comes in discrete steps labeled by integers. A world whose evolution is discrete could be called *stroboscopic*.

A system that changes with time is called a *dynamical system*. A dynamical system consists of more than a space of states. It also entails a *law of motion*, or *dynamical law*. The dynamical law is a rule that tells us the next state given the current state.

One very simple dynamical law is that whatever the state at some instant, the next state is the same. In the case of our example, it has two possible histories: H H H H H H . . . and T T T T T T . . . .

Another dynamical law dictates that whatever the current state, the next state is the opposite. We can make diagrams to illustrate these two laws. Figure 2 illustrates the first law, where the arrow from H goes to H and the arrow from T goes to T. Once again it is easy to predict the future: If you start with H, the system stays H; if you start with T, the system stays T.

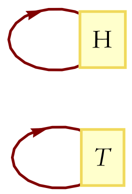

Figure 2: A dynamical law for a two-state system.

A diagram for the second possible law is shown in Figure 3, where the arrows lead from H to T and from T to H. You can still predict the future. For example, if you start with H the history will be H T H T H T H T H T . . . . If you start with T the history is T H T H T H T H . . . .

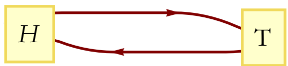

Figure 3: Another dynamical law for a two-state system.

We can even write these dynamical laws in equation form. The variables describing a system are called its *degrees of freedom*.

Our coin has one degree of freedom, which we can denote by the greek letter sigma, σ. Sigma has only two possible values; $\sigma = 1$ and $\sigma = -1$, respectively, for H and T. We also use a symbol to keep track of the time. When we are considering a continuous evolution in time, we can symbolize it with t. Here we have a discrete evolution and will use n. The state at time n is described by the symbol $\sigma(n)$, which stands for σ at n.

Let's write equations of evolution for the two laws. The first law says that no change takes place. In equation form,

$$\sigma(n+1) = \sigma(n) .$$

In other words, whatever the value of σ at the nth step, it will have the same value at the next step.

The second equation of evolution has the form

$$\sigma(n+1) = -\sigma(n),$$

implying that the state flips during each step.

Because in each case the future behavior is completely determined by the initial state, such laws are deterministic. All the basic laws of classical mechanics are deterministic.

To make things more interesting, let's generalize the system by increasing the number of states. Instead of a coin, we could use a six-sided die, where we have six possible states (see Figure 4).

Now there are a great many possible laws, and they are not so easy to describe in words—or even in equations. The simplest way is to stick to diagrams such as Figure 5. Figure 5 says that given the numerical state of the die at time n, we increase the state one unit at the next instant $n + 1$. That works fine until we get to 6, at which point the diagram tells you to go back to 1 and repeat the pattern. Such a pattern that is repeated endlessly is called a *cycle*. For example, if we start with 3 then the history is 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, ... We'll call this pattern Dynamical Law 1.

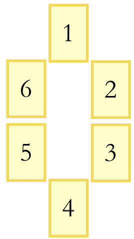

Figure 4: A six-state system.

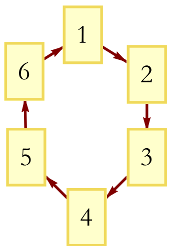

Figure 5: Dynamical Law 1.

Figure 6 shows another law, Dynamical Law 2. It looks a little messier than the last case, but it's logically identical—in each case the system endlessly cycles through the six possibilities. If we relabel the states, Dynamical Law 2 becomes identical to Dynamical Law 1.

Not all laws are logically the same. Consider, for example, the law shown in Figure 7. Dynamical Law 3 has two cycles. If you start on one of them, you can't get to the other. Nevertheless, this law is completely deterministic. Wherever you start, the future is determined. For example, if you start at 2, the history will be 2, 6, 1, 2, 6, 1, . . . and you will never get to 5. If you start at 5 the history is 5, 3, 4, 5, 3, 4, . . . and you will never get to 6.

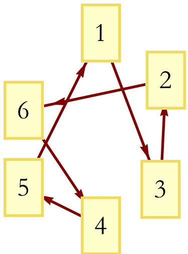

Figure 6: Dynamical Law 2.

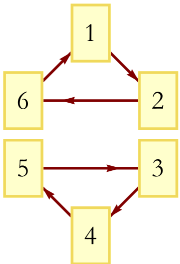

Figure 7: Dynamical Law 3.

Figure 8 shows Dynamical Law 4 with three cycles.

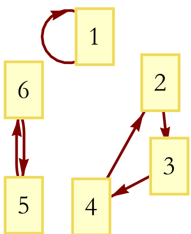

Figure 8: Dynamical Law 4.

It would take a long time to write out all of the possible dynamical laws for a six-state system.

Exercise 2: Can you think of a general way to classify the laws that are possible for a six-state system?

Rules That Are Not Allowed: The Minus-First Law

According to the rules of classical physics, not all laws are legal. It's not enough for a dynamical law to be deterministic; it must also be reversible.

The meaning of *reversible*—in the context of physics—can be described a few different ways. The most concise description is to say that if you reverse all the arrows, the resulting law is still deterministic. Another way, is to say *the laws are deterministic into the past as well as the future*. Recall Laplace's remark, "for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes." Can one conceive of laws that are deterministic into the future, but not into the past? In other words, can we formulate irreversible laws? Indeed we can. Consider Figure 9.

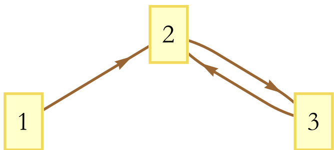

Figure 9: A system that is irreversible.

The law of Figure 9 does tell you, wherever you are, where to go next. If you are at 1, go to 2. If at 2, go to 3. If at 3, go to 2.

There is no ambiguity about the future. But the past is a different matter. Suppose you are at 2. Where were you just before that? You could have come from 3 or from 1. The diagram just does not tell you. Even worse, in terms of reversibility, there is no state that leads to 1; state 1 has no past. The law of Figure 9 is *irreversible*. It illustrates just the kind of situation that is prohibited by the principles of classical physics.

Notice that if you reverse the arrows in Figure 9 to give Figure 10, the corresponding law fails to tell you where to go in the future.

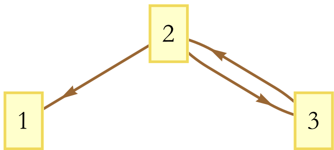

Figure 10: A system that is not deterministic into the future.

There is a very simple rule to tell when a diagram represents a deterministic reversible law. If every state has a single unique arrow leading into it, and a single arrow leading out of it, then it is a legal deterministic reversible law. Here is a slogan: *There must be one arrow to tell you where you're going and one to tell you where you came from.*

The rule that dynamical laws must be deterministic and reversible is so central to classical physics that we sometimes forget to mention it when teaching the subject. In fact, it doesn't even have a name. We could call it the first law, but unfortunately there are already two first laws—Newton's and the first law of thermodynamics. There is evan a zeroth law of thermodynamics. So we have to go back to a *minus-first law* to gain priority for what is undoubtedly the most fundamental of all physical laws—*the conservation of information*. The conservation of information is simply the rule that every state has one arrow in and one arrow out. It ensures that you never lose track of where you started.

The conservation of information is not a conventional conservation law. We will return to conservation laws after a digression into systems with infinitely many states.

Dynamical Systems with an Infinite Number of States

So far, all our examples have had state-spaces with only a finite number of states. There is no reason why you can't have a dynamical system with an infinite number of states. For example, imagine a line with an infinite number of discrete points along it—like a train track with an infinite sequence of stations in both directions. Suppose that a marker of some sort can jump from one point to another according to some rule. To describe such a system, we can label the points along the line by integers the same way we labeled the discrete instants of time above. Because we have already used the notation n for the discrete time steps, let's use an uppercase N for points on the track. A history of the marker would consist of a function $N(n)$, telling you the place along the track N at every time n. A short portion of this state-space is shown in Figure 11.

Figure 11: State-space for an infinite system.

A very simple dynamical law for such a system, shown in Figure 12, is to shift the marker one unit in the positive direction at each time step.

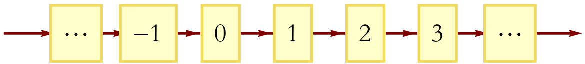

Figure 12: A dynamical rule for an infinite system.

This is allowable because each state has one arrow in and one arrow out. We can easily express this rule in the form of an equation.

$$N(n+1) = N(n) + 1 \tag{1}$$

Here are some other possible rules, but not all are allowable.

$$N(n+1) = N(n) - 1 \tag{2}$$

$$N(n+1) = N(n) + 2 \tag{3}$$

$$N(n+1) = N(n)^2 \tag{4}$$

$$N(n+1) = -1^{N(n)} N(n) \tag{5}$$

> **Exercise 3: Determine which of the dynamical laws shown in Eq.s (2) through (5) are allowable.**

In Eq. (1), wherever you start, you will eventually get to every other point by either going to the future or going to the past. We say that there is a single infinite cycle. With Eq. (3), on the other hand, if you start at an odd value of N, you will never get to an even value, and vice versa. Thus we say there are two infinite cycles.

We can also add qualitatively different states to the system to create more cycles, as shown in Figure 13.