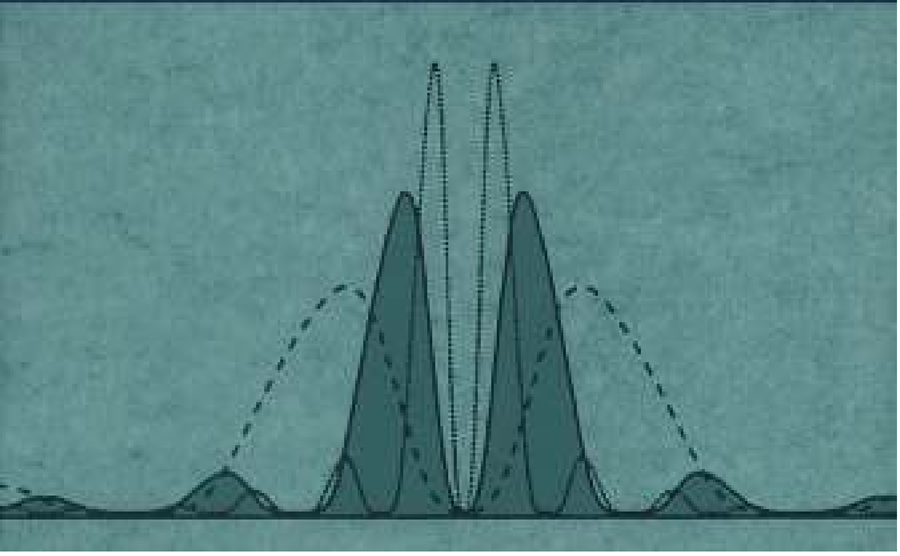
Ali N. Akansu / Richard A. Haddad

Second Edition

MULTIRESOLUTION SIGNAL DECOMPOSITION

Transforms · Subbands · Wavelets



Multiresolution Signal Decomposition

Transforms, Subbands, and Wavelets

Second Edition

Series in Telecommunications

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Multiresolution Signal Decomposition: Transforms, Subbands, and Wavelets
Ali N. Akansu and Richard A. Haddad

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To Bilge and Elizabeth

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Preface

Since the first edition of this book in 1992 we have witnessed a flood of books, texts, monographs, and edited volumes describing different aspects of block transforms, multirate filter banks, and wavelets. Some of these have been mathematically precise, designed for the rigorous theoretician, while others sought to interpret work in these areas for engineers and students.

The field is now mature, yet active. The theory is much better understood in the signal processing community, and applications of the multiresolution concept to situations in digital multimedia, communications, and others abound. In the first edition and in the early days of multirate filter banks a prime emphasis was on signal compaction and coding. Today, multiresolution decomposition and time-frequency concepts have opened up new vistas for further development and applications. These ideas concerning orthogonal signal analysis and synthesis have led to applications in digital audio broadcasting, digital data hiding and watermarking, wireless and wireline communications, audio and video coding, and many others.

In this edition, we continue to treat block transforms, subband filter banks, and wavelets from a common, unifying standpoint. We demonstrate the commonality among these signal analysis and synthesis techniques by showing how the block transform evolves gradually into the more general multirate subband filter bank, and then by establishing the multiresolution decomposition features common to both the dyadic subband tree structure and the orthonormal wavelet transform. In order to achieve this unification, we have focused mainly on *orthonormal* decompositions and presented a unified and integrated treatment of multiresolution signal decomposition techniques using the property of orthonormality as the unifying theme. (A few exceptions, such as the oversampled Laplacian pyramid and biorthogonal filter banks are also presented because they provide an historical perspective and serve as foils to the critically sampled, orthonormal subband structures we emphasize.)

Our second focus in the first edition was the application of decomposition techniques to signal *compression* and coding. Accordingly, we describe *objective* performance criteria that measure this attribute and then compare the different techniques on this basis. We acknowledge that subjective evaluations of decompositions are important in applications such as image and video processing and coding, machine vision, and pattern recognition. Such aspects are treated adequately in the literature cited and are deemed beyond the scope of this book. A new focus in this edition is the time-frequency properties of signals and decomposition techniques. Accordingly, this text provides tables listing the coefficients of popular block transforms, subband and wavelet filters, and also their time-frequency features and compaction performance for both theoretical signal models and standard test images. In this respect, we have tried to make the book a reference text as well as a didactic monograph.

Our approach is to build from the fundamentals, taking simple representative cases first and then extending these to the next level of generalization. For example, we start with block transforms, extend these to lapped orthogonal transforms, and then show both to be special cases of subband filter structures. We have avoided the theorem-proof approach, preferring to give explanation and derivations emphasizing clarity of concept rather than strict rigor.

Chapter 2 on orthogonal transforms introduces block transforms from a least-squares expansion in orthogonal functions. Signal models and decorrelation and compaction performance measures are then used to evaluate and compare several proposed block and lapped transforms. The biorthogonal signal decomposition is mentioned.

Chapter 3 presents the theory of perfect reconstruction, orthonormal two-band and M-band filter banks with emphasis on the finite impulse response variety. A key contribution here is the time-domain representation of an arbitrary multirate filter bank, from which a variety of special cases emerge—paraunitary, biorthogonal, lattice, LOT, and modulated filter banks. The two-channel, dyadic tree structure then provides a multiresolution link with both the historical Laplacian pyramid and the orthonormal wavelets of Chapter 6. A new feature is the representation of the transmultiplexer as the synthesis/analysis dual of the analysis/synthesis multirate filter bank configuration.

Chapter 4 deals with specific filter banks and evaluates their objective performance. This chapter relates the theory of signal decomposition techniques presented in the text with the applications. It provides a unified performance evaluation of block transforms, subband decomposition, and wavelet filters from a signal processing and coding point of view. The topic of optimal filter banks presented in this chapter deals with solutions based on practical considerations in image coding. The chapter closes with the modeling and optimum design of quantized filter banks.

Chapter 5 on time-frequency (T-F) focuses on joint time-frequency properties of signals and the localization features of decomposition tools. There is a discussion of techniques for synthesizing signals and block transforms with desirable T-F properties and describes applications to compaction and interference excision in spread spectrum communications.

Chapter 6 presents the basic theory of the orthonormal and biorthogonal wavelet transforms and demonstrates their connection to the orthonormal dyadic subband tree of Chapter 3. Again our interest is in the linkage to the multiresolution subband tree structure, rather than with specific applications of wavelet transforms.

Chapter 7 is a review of recent applications of these techniques to image coding, and to communications applications such as discrete multitone (DMT) modulation, and orthogonal spread spectrum user codes. This chapter links the riches of linear orthogonal transform theory to the popular and emerging transform applications. It is expected that this linkage might spark ideas for new applications that benefit from these signal processing tools in the future.

This book is intended for graduate students and R&D practitioners who have a working knowledge of linear system theory and Fourier analysis, some linear algebra, random signals and processes, and an introductory course in digital signal processing. A set of problems is included for instructional purposes.

For classroom presentation, an instructor may present the material in the text in three packets:

(1) Chapters 2 and 5 on block transforms and time-frequency methods
(2) Chapters 3 and 4 on theory and design of multirate filter banks
(3) Chapters 6 and 7 on wavelets and transform applications

As expected, a book of this kind would be impossible without the cooperation of colleagues in the field. The paper preprints, reports, and private communications they provided helped to improve significantly the quality and timeliness of the book. We acknowledge the generous help of N. Sezgin and A. Bircan for some figures. Dr. T. Russell Hsing of Bellcore was instrumental in introducing us to Academic Press. It has been a pleasure to work with Dr. Zvi Ruder during this project. Dr. Eric Viscito was very kind to review Chapter 3. The comments and suggestions of our former and current graduate students helped to improve the quality of this book. In particular, we enjoyed the stimulating discussions and interactions with H. Caglar, A. Benyassine, M. Tazebay, X. Lin, N. Tanrı, K. Park, K. Kwak, and G. Hung. We thank them all. Lastly, we appreciate and thank

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Ali N. Akansu
Richard A. Haddad
April 2000

Chapter 1

Introduction

1.1 Introduction

In the first edition of this book, published in 1992, we stated our goals as threefold:
(1) To present orthonormal signal decomposition techniques—transforms, subbands, and wavelets—from a unified framework and point of view.
(2) To develop the interrelationships among decomposition methods in both time and frequency domains and to define common features.
(3) To evaluate and critique proposed decomposition strategies from a compression/coding standpoint using measures appropriate to image processing.
The emphasis then was signal coding in an analysis/synthesis structure or codec. As the field matured and new insights were gained, we expanded our vistas to communications systems and other applications where objectives other than compression are vital — as for example, interference excision in CDMA spread spectrum systems. We can also represent certain communications systems such as TDMA, FDMA, and CDMA as synthesis/analysis structures, i.e., the conceptual dual of the compression codec. This duality enables one to view all these systems from one unified framework.

The Fourier transform and its extensions have historically been the prime vehicle for signal analysis and representation. Since the early 1970s, block transforms with real basis functions, particularly the discrete cosine transform (DCT), have been studied extensively for transform coding applications. The availability of simple fast transform algorithms and good signal coding performance made the DCT the standard signal decomposition technique, particularly for image and video. The international standard image-video coding algorithms, i.e., CCITT

H.261, JPEG, and MPEG, all employ DCT-based transform coding.

Since the recent research activities in signal decomposition are basically driven by visual signal processing and coding applications, the properties of the human visual system (HVS) are examined and incorporated in the signal decomposition step. It has been reported that the HVS inherently performs multiresolution signal processing. This finding triggered significant interest in multiresolution signal decomposition and its mathematical foundations in multirate signal processing theory. The multiresolution signal analysis concept also fits a wide spectrum of visual signal processing and visual communications applications. Lower- or coarser-resolution versions of an image frame or video sequence are often sufficient in many instances. Progressive improvement of the signal quality in visual applications, from coarse to finer resolution, has many uses in computer vision, visual communications, and related fields.

The recognition that multiresolution signal decomposition is a by-product of multirate subband filter banks generated significant interest in the design of better performing filter banks for visual signal processing applications.

The wavelet transform with a capability for variable time-frequency resolution has been promoted as an elegant multiresolution signal processing tool. It was shown that this decomposition technique is strongly linked to subband decomposition. This linkage stimulated additional interest in subband filter banks, since they serve as the only vehicle for fast orthonormal wavelet transform algorithms and wavelet transform basis design.

1.2 Why Signal Decomposition?

The uneven distribution of signal energy in the frequency domain has made signal decomposition an important practical problem. Rate-distortion theory shows that the uneven spectral nature of real-world signals can provide the basis for source compression techniques. The basic concept here is to divide the signal spectrum into its subspectra or subbands, and then to treat those subspectra individually for the purpose at hand. From a signal coding standpoint, it can be appreciated that subspectra with more energy content deserve higher priority or weight for further processing. For example, a slowly varying signal will have predominantly low-frequency components. Therefore, the low-pass subband contains most of its total energy. If one discards the high-pass analysis subbands and reconstructs the signal, it is expected that very little or negligible reconstruction error occurs after this analysis-synthesis operation.

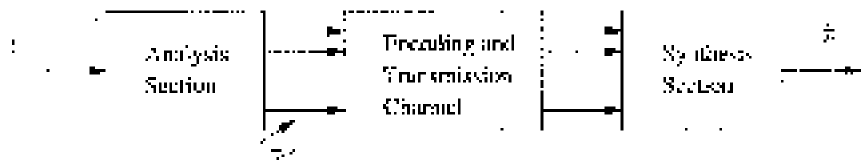
The decomposition of the signal spectrum into subbands provides the mathematical basis for two important and desirable features in signal analysis and processing. First, the monitoring of signal energy components within the subbands or subspectra is possible. The subband signals can then be ranked and processed independently. A common use of this feature is in the spectral shaping of quantization noise in signal coding applications. By bit allocation we can allow different levels of quantization error in different subbands. Second, the subband decomposition of the signal spectrum leads naturally to multiresolution signal decomposition via multirate signal processing in accordance with the Nyquist sampling theorem.

Apart from coding/compression considerations, signal decomposition into subbands permits us to investigate the subbands for contraband signals, such as band-limited or single-tone interference. We have also learned to think more globally (to the point of signal decomposition in a composite time-frequency domain, rather than in frequency subbands as such. This expansive way of thinking leads naturally to the concept of wavelet packets (subband trees), and to the block transform packets introduced in this text.

1.3 Decompositions: Transforms, Subbands, and Wavelets

The signal decomposition (and reconstruction) techniques developed in this book have three salient characteristics:
(1) Orthonormality. As we shall see, the block transforms will be square unitary matrices, i.e., the rows of the transformation matrix will be orthogonal to each other; the subband filter banks will be *paraunitary*, a special kind of orthonormality; and the wavelets will be orthonormal.
(2) Perfect reconstruction (PR). This means that, in the absence of encoding, quantization, and transmission errors, the reconstructed signal can be reassembled perfectly at the receiver.
(3) Critical sampling. This implies that the signal is subsampled at a minimum possible rate consistent with the applicable Nyquist theorem. From a practical standpoint, this means that if the original signal has a data rate of f_s samples or pixels per second, the sum of the transmission rates out of all the subbands is also f_s.

The abovementioned are the prime ingredients of the decomposition techniques. However, we also briefly present a few other decomposition methods for contrast or historical perspective. The oversampled Laplacian pyramid, biorthogonal filter banks, and non-PR filter banks are examples of these, which we introduce for didactic value.



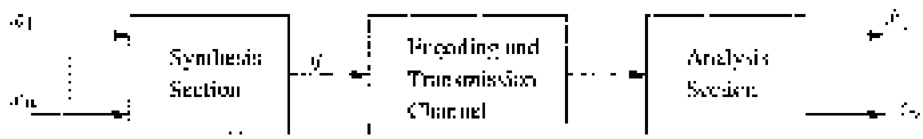


Figure 1.1: (a) Analysis-synthesis structure. (b) synthesis/analysis system.

As shown in Fig. 1.1(a), the input signal x is decomposed in the analysis section, encoded, and transmitted. At the receiver or synthesis section, it is reconstructed as $\hat{x}$. In a perfect reconstruction system $\hat{x} = x$ within an allowable delay. In a critically sampled system, the sum of the data rates of the decomposed signal components equals that of the input signal.

In Fig. 1.1(b), the dual operation is shown. Typically, the synthesis section could be a TDMA or FDMA multiplexer wherein several signals are separated in time (TDMA), frequency (FDMA), or in time-frequency (CDMA), and combined into one signal for transmission. The received signal is then separated into components in the analysis section.

1.3.1 Block Transforms and Filter Banks

In block transform notation, the analysis or decomposition operation suggested in Fig. 1.1 is done with a blockwise treatment of the signal. The input signal is first segmented into nonoverlapping blocks of samples. These signal blocks or vectors are transformed into spectral coefficient vectors by the orthogonal matrix. The spectral unevenness of the signal is manifested by unequal coefficient energies by this technique and only transform coefficients with significant energies need be considered for further processing. Block transforms, particularly the discrete cosine transforms, have been used in image-video coding. Chapter 2 introduces and discusses block transforms in detail and provides objective performance evaluations of known block transforms. The Karhunen-Loeve transform, or KLT, is the unique input-signal dependent optimal block transform. We derive its properties and use it as a standard against which all other fixed transforms can be compared.

In block transforms, the duration or length of the basis functions is equal to the size of the data block. This implies that the transform and inverse transform matrices are square. This structure has the least possible freedom in tuning its basis functions. It can meet only an orthonormality requirement and, for the optimal KLT, generate uncorrelated spectral coefficients. Limited joint time–frequency localization of basis functions is possible using the concept of block transform packets (Chapter 5).

More freedom for tuning the basis functions is possible if we extend the duration of these functions. Now this rectangular transform or decomposition has overlapping basis functions. This overlapping eliminates the "blockiness" problem inherent in block transforms. Doubling the length of the basis sequences gives the lapped orthogonal transform, or LOT, as discussed in Section 2.5.

In general, if we allow arbitrary durations for the basis sequence filters, the finite impulse response (FIR) filter bank or subband concept is reached. Therefore, block transforms and LOTs can be regarded as special filter banks. The multirate signal processing theory and its use in perfect reconstruction analysis-synthesis filter banks are discussed in depth in Chapter 3. This provides the common frame through which block transforms, LOTs, and filter banks can be viewed.

Figure 1.2 shows a hierarchical conceptual framework for viewing these ideas. At the lowest level, the block transform is a bank of M filters whose impulse responses are of length $L = M$. At the next level, the LOT is a bank of M filters, each with impulse responses (or basis sequences) of length $L = 2M$. At the top of the structure is the M-band multirate filter bank with impulse responses of any length $L > M$. On top of that is the M-band multirate filter bank with impulse responses of arbitrary length $L \geq M$. This subband structure is illustrated in Fig. 1.3(a), where the signal is decomposed into M equal bands by the filter bank.

The filter bank often used here has frequency responses covering the M bands from 0 to $f_s/2$. When these frequency responses are translated versions of a low-frequency prototype, the bank is called a modulated filter bank.

The Nyquist theorem in a multiband system can now be invoked to subsample each band. The system is critically subsampled (or maximally decimated) when the decimation factor D or subsampling parameter equals the number of subbands M. When $D < M$, the system is oversampled.

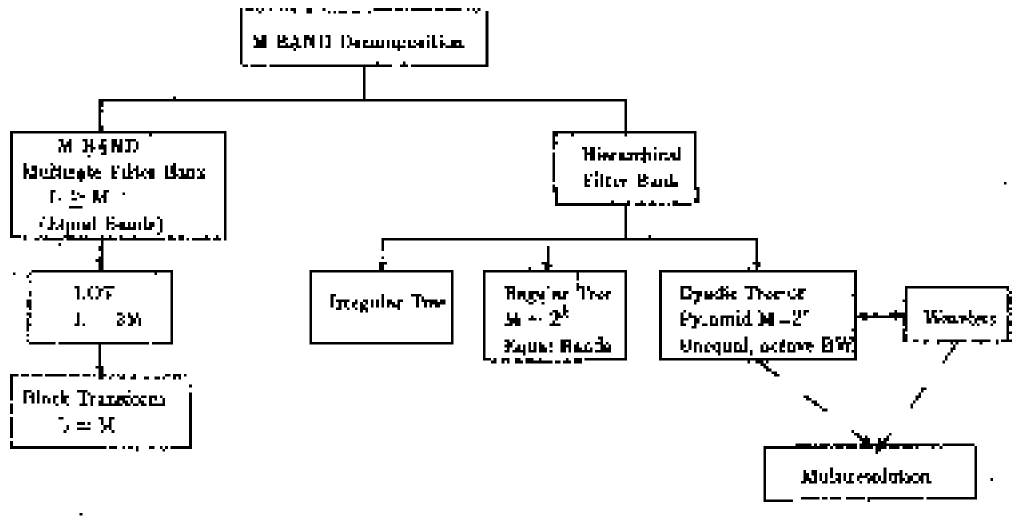


Figure 1.2: An overview of M-band signal decomposition.

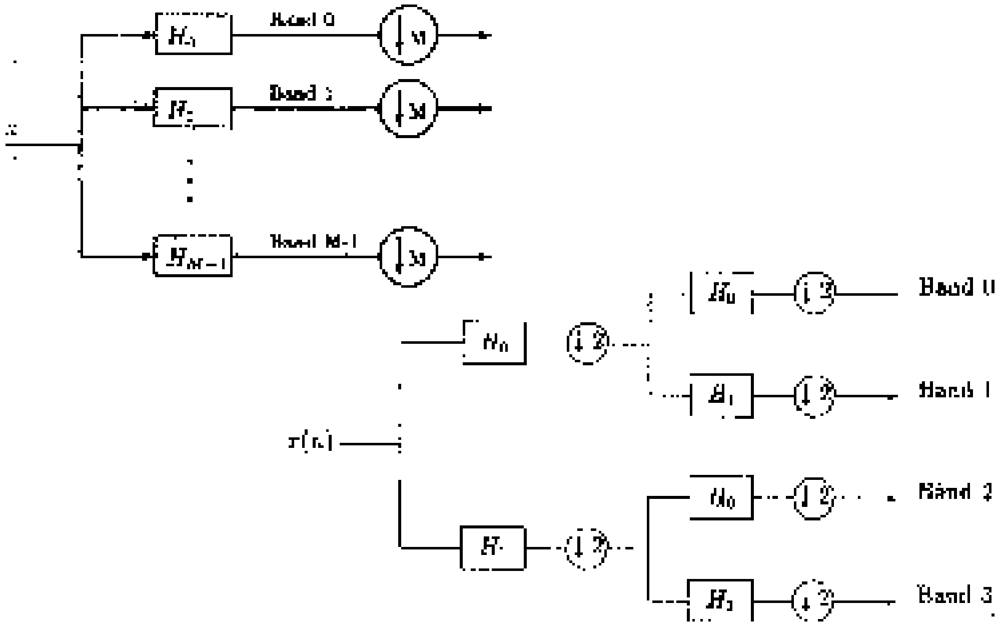


Figure 1.3: Multirate filter bank with equal bandwidths: (a) M-band; (b) four band realized by a two-level binary (regular) tree.

Another way of realizing the decomposition into M equal subbands is shown by the hierarchical, two-band subband tree shown in Fig. 1.3(b). Each level of the tree splits the preceding subband into two equal parts, permitting a decomposition into $M = 2^K$ equal subbands. In this case the M-band structure is said to be realized by a dilation of the impulse response of the basic two-band structure at each level of the tree since splitting each subband in two dilates the impulse response by this factor.

1.3.2 Multiresolution Structures

Yet another possible decomposition is shown in Fig. 1.4, which represents a "dyadic tree" decomposition. The signal is first split into low- and high-frequency components in the first level. This first low-frequency subband, containing most of the energy, is subsampled and again decomposed into low- and high-frequency subbands. This process can be continued into K levels. The coarsest signal is the one labeled LLL in the figure. Moving from right to left in this diagram, we see a progression from coarser to finer signal representation as the high-frequency "detail" is added at each level. The signal can thus be approximately represented by different resolutions at each level of the tree.

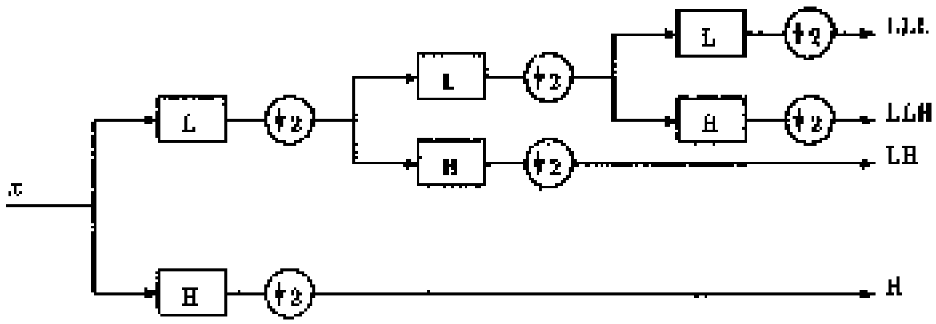


Figure 1.4: Multiresolution dyadic tree. L and H represent low-pass and high-pass filters, respectively.

An oversampled version of this tree, called the Laplacian pyramid, was first introduced for image coding by Burt and Adelson (1983). These topics are explained in detail and the reference is given in Chapter 3.

Wavelet transforms recently have been proposed as a new multiresolution decomposition tool for continuous-time signals. The kernel of the wavelet transform is obtained by dilation and translation of a prototype bandpass function. The