Math for Life

Crucial Ideas You Didn't Learn in School

UPDATED EDITION

What are our *real* choices for energy?

How do politicians play games with redistricting?

Is it safe to talk and drive?

— and much more

Jeffrey Bennett

- How can we solve the national debt crisis?
- Should you or your child take on a student loan?
- Is it safe to talk on a cell phone while driving?
- Are there viable energy alternatives to fossil fuels?
- What could you do with a billion dollars?
- Could simple policy changes reduce political polarization?

These questions may all seem very different, but they share two things in common. First, they are a questions with important implications for either personal success or our success as a nation. Second, th all concern topics that we can fully understand only with the aid of clear quantitative or mathematic thinking. In other words, they are topics for which we need *math for life*—a kind of math that loo quite different from most of the math that we learn in school, but that is just as (and often mor important.

In *Math for Life,* award-winning author Jeffrey Bennett simply and clearly explains the key ideas quantitative reasoning and applies them to all the above questions and many more. He also uses the questions to analyze our current education system, identifying both shortfalls in the teaching mathematics and solutions for our educational future.

No matter what your own level of mathematical ability, and no matter whether you approach th book as an educator, student, or interested adult, you are sure to find something new and though provoking in *Math for Life.*

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Preface

The housing bubble. Lotteries. Cell phones and driving. Personal budgeting. The federal debt. Soci[al] Security. Tax reform. Energy policy. Global warming. Political redistricting. Population growth[.] Radiation from nuclear power plants.

What do all the above have in common? Each is a topic with important implications for all of us, b[ut] also a topic that we can fully understand only if we approach it with clear quantitative or mathematic[al] thinking. In other words, these are all topics for which we need "math for life"—a kind of math th[at] looks quite different from most of the math that we learn in school, but that is just as (and sometim[es] more) important.

Now, in case the word "math" has you worried for any reason, rest assured that this is not a mat[h] book in any traditional sense. You won't find any complex equations in this book, nor will you se[e] anything that looks much like what you might have studied in high school or college mathemati[cs] classes. Instead, the focus of this book will be on what is sometimes called *quantitative reasoning,* whic[h] means using numbers and other mathematically based ideas to reason our way through the kinds [of] problems that confront us in everyday life. As the list in the first paragraph should show, these proble[ms] range from the personal to the global, and over everything in between.

So what exactly will you learn about "math for life" in this short book? Perhaps the best way for m[e] to explain it is to list my three major goals in writing this book:

1. On a personal level, I hope this book will prove *practical* in helping you make decisions that will improve your health, yo[ur] happiness, and your financial future. To this end, I'll discuss some general principles of quantitative reasoning that you may n[ot] have learned previously, while also covering specific examples that will include how to evaluate claims of health benefits that y[ou] may hear in the news (or in advertisements) and how to make financial decisions that will keep you in control of your own life.

2. On a societal level, I hope to draw attention to what I believe are oft-neglected mathematical truths that underlie many of the m[ost] important problems of our time. For example, I believe that far too few of us (and far too few politicians) understand the tr[ue] magnitude of our current national budget predicament, the true challenge of meeting our future energy needs, or what it means [to] live in a world whose population may increase by another 3 billion people during the next few decades. I hope to show you how [a] little bit of quantitative reasoning can illuminate these and other issues, thereby making it more likely that we'll find ways to brid[ge] the political differences that have up until now stood in the way of real solutions.

3. On the level of educational policy, I hope that this book will have an impact on the way we think about mathematics education. [As] I'll argue throughout the book, I believe that we can and must do a much better job both in teaching our children tradition[al] mathematics—meaning the kind of mathematics that is necessary for modern, high-tech careers—and in teaching the mathemat[ics] of quantitative reasoning that we all need as citizens in today's society. I'll discuss both the problems that exist in our curre[nt] educational system and the ways in which I believe we can solve them.

With those three major goals in mind, I'll give you a brief overview of how I've structured the boo[k.] The first chapter focuses on the general impact of societal attitudes toward math. In particular, I['ll] explain why I think the fact that so many people will without embarrassment say that they are "bad [at] math" was a major contributing factor to the housing bubble and the recent recession; I'll also discu[ss] the roots of poor attitudes toward math and how we can change those attitudes in the future. Th[e] second and third chapters provide general guidance for understanding the kinds of mathematical a[nd] statistical thinking that lie at the heart of many modern issues and that are in essence the core concep[ts] of "math for life." The remaining chapters are topic-based, covering all the issues I listed above, an[d] more; note that, while I'd like to think you'll read the book cover to cover, I've tried to make th[e] individual chapters self-contained enough so that you could read them in any order. Finally, in th[e]

epilogue, I'll offer my personal suggestions for changing the way we approach and teach mathematics.

As an author, I always realize that readers are what make my work possible, and I thank you for taking the time to at least have a look at this book. If I've convinced you to read it through, I hope you will find it both enjoyable and useful.

Jeffrey Bennett
Boulder, Colorado

(Don't Be)
"Bad at Math"

Nothing in life is to be feared. It is only to be understood.
— Marie Curie

Equations are just the boring part of mathematics.
— Stephen Hawking

Let's start with a multiple-choice question.

Question: Imagine that you're at a party, and you've just struck up a conversation with a dynamic, successful businesswoman. Which of the following are you most likely to hear her say during the course of your conversation?

Answer choices:

a. "I really don't know how to read very well."

b. "I can't write a grammatically correct sentence."

c. "I'm awful at dealing with people."

d. "I've never been able to think logically."

e. "I'm bad at math."

We all know that the answer is E, because we've heard it so many times. Not just from businesswomen and businessmen, but from actors and athletes, construction workers and sales clerks, and sometimes even teachers and CEOs. Somehow, we have come to live in a society in which many otherwise successful people not only have a problem with mathematics but are unafraid to admit it. In fact, it's sometimes stated almost as a point of pride, with little hint of embarrassment.

It doesn't take a lot of thought to realize that this creates major problems. Mathematics underlies nearly everything in modern society, from the daily financial decisions that all of us must make to the way in which we understand and approach global issues of the economy, politics, and science. We cannot possibly hope to act wisely if we don't have the ability to think critically about mathematical ideas.

This fact takes us immediately to one of the main themes of this book. Look again at our opening multiple-choice question. It would be difficult to imagine the successful businesswoman admitting to any of choices A through D, even if they were true, because all would be considered marks of ignorance and shame. I hope to convince you that choice E should be equally unacceptable. Through numerous examples, I will show you ways in which being "bad at math" is exacting a high toll on individuals, on our nation, and on our world. Along the way, I'll try to offer insights into how we can learn to make

better decisions about mathematically based issues. I hope the book will thereby be of use to everyon~~but it's especially directed at those of you who might currently think of yourselves as "bad at math~~ With luck, by the time you finish reading, you'll have a very different perspective both on the importan of mathematics and on your own ability to understand it.

Of course, I can't turn you into a mathematician in a couple hundred pages, and a quick scan of th book should relieve you of any fear that I'm expecting you to repeat the kinds of equation solving th you may remember from past math classes. Instead, this book contains a type of math that you actual *need* for life in the modern world, but which you probably were never taught before.

Best of all, this is a type of mathematics that anyone can learn. You don't have to be a whiz calculations, or know how to solve calculus equations. You don't need to remember the quadrat formula, or most of the other facts that you were expected to memorize in high school algebra. All yo need to do is open your mind to new ways of thinking that will enable you to reason as clearly wi numbers and ideas of mathematics as you do without them.

The Math Recession

For our first example, let's consider the recent Great Recession, which left millions of people unemploye stripped millions of others of much of their life savings, and pushed the global financial system so clo to collapse that governments came in with hundreds of billions of dollars in bailout funds. The clea trigger for the recession was the popping of the real estate bubble, which ignited a mortgage crisis. B what created the bubble that popped? I believe a large part of the answer can be traced to po mathematical thinking.

Take a look at Figure 1, which shows one way of looking at home prices during the past few decad The bump starting in 2001 represents the housing price bubble. Let's use some quantitative reasoning t see why it should have been obvious that the bubble was not sustainable.

Figure 1. Data used with permission of the Joint Center for Housing Studies of Harvard University. All rights reserved.

Here's how to think about it. As its title indicates, the graph shows the ratio of the average (median) home price to the average income. For example, if the average household income were $50,000 per year, then a ratio of 3.0 would mean that the average home price was three times the average income, or $150,000. The graph shows that the average ratio for the three decades prior to the start of the bubble was actually about 3.2, which means someone with an income of $50,000 typically purchased a house costing about $160,000 (which you find by multiplying 3.2 by $50,000).

Now look at what happened during the housing bubble. After increasing modestly in the 1990s, the ratio began shooting upward in 2001, reaching a peak of about 4.7 in 2005. This was nearly a 50% increase from the historical average of 3.2, which means that relative to income, the average home was about 50% more expensive in 2005 than it was before the bubble. In other words, a family that previously would have bought a house costing $160,000 was instead buying one that cost nearly $240,000.

With homes so much more expensive relative to income, families had to spend a higher percentage of their income on them. In general, a family can spend a higher percentage of its income on housing only if some combination of the following three things happens: (1) its income increases; (2) it cuts expenses in other areas; or (3) it borrows more money. Other statistics showed clearly that average income was *not* rising significantly, and that while homeowners gained some benefit from relatively low mortgage interest rates, overall consumer spending actually increased. We are therefore left with the third possibility: that the housing bubble was fueled primarily by borrowing. With little prospect that income would rise dramatically in the future, it was inevitable that this borrowing would be unaffordable and that loan defaults and foreclosures would follow. The only way to restore equilibrium to the system was for home prices to fall dramatically.

Lest you think that this is a case of hindsight being 20/20, keep in mind that these kinds of data were available throughout the growth of the bubble. Anyone willing to think about it should therefore have known that the bubble would inevitably pop, and, indeed, you can find many articles from the time that pointed out this obvious fact. So how did everyone else manage to miss it?

Although it's tempting to blame the problem on a failure of "the system," it was ultimately the result of millions of individual decisions, most of which involved a real estate agent arguing that prices could only go up, a mortgage broker offering an unaffordable loan, and a customer buying into the real estate hype while ignoring the fact that the mortgage payments would become outsized relative to his or her income. In short, many of us ignored the mathematical reality staring us in the face.

That is why I think of the Great Recession as a "math recession": It was caused by the fact that too many of us were unwilling or unable to think mathematically. Perhaps I'm overly idealistic, but I believe that with better math education—and especially with more emphasis on quantitative reasoning—many more people would have questioned the bubble before it got out of hand. We can't change the past, but I hope this lesson will convince you that we all need to get over being "bad at math."

Fear and Loathing of Mathematics

If we as a society (or you as an individual) are going to overcome the problems caused by being "bad at math," a first step is understanding why this form of ignorance has become socially acceptable. This social acceptance is not as natural as it might seem, and in fact is relatively rare outside the United States. Research has shown that infants have innate mathematical capabilities, and it's difficult to find

kindergartners who don't get a thrill out of seeing how high they can count; both facts suggest that mo[st] of us are born with an affinity for mathematics. Even many adults who proclaim they are "bad at" math[...] must once have been quite good at it. After all, the successful businesswoman of our multiple-choi[ce] question probably could not have gotten where she is without decent grades.

My own attempt to understand the origins of the social acceptance of "bad at math" began wit[h] surveys of students who took a course in quantitative reasoning that I developed and taught at t[he] University of Colorado. This course was designed specifically for students who did not plan to take an[y] other mathematics courses in college, and the only reason they took this one was because they needed [it] to fulfill a graduation requirement. In other words, it was filled with students who had already decide[d] that math wasn't for them. When asked why, the students divided themselves roughly into two group[s] which I call *math phobics* and *math loathers*. The math phobics generally did poorly in their high scho[ol] mathematics classes and therefore came to fear the subject. The math loathers actually did pretty well [in] high school math but still ended up hating it.[1]

Probing further, I asked students to try to recall where their fear or loathing of mathematics ma[y] have originated. Interestingly, the most common responses traced these attitudes to one or a fe[w] particular experiences in elementary or secondary school. Many of the students said they had like[d] mathematics until one adult, often a teacher but sometimes a parent or a family friend, did somethin[g] that turned them off, such as telling the student that he or she was no good at math, or laughing at th[e] student for an incorrect solution. Dismayingly, women were far more likely to report such experience[s] than men. Apparently, it is still quite common for girls as young as elementary age to be told that, ju[st] because they are girls, they can't be any good at math.

Who would say such things to young children, thereby afflicting them with a lifelong fear or loathin[g] of mathematics? Certainly, there are cases where the offending adult is a math teacher with some sort [of] superiority complex. But more commonly, it appears that the adults who turn kids off from mathemati[cs] are those who are themselves afflicted with the "bad at math" syndrome. Like an infectious disease, "ba[d] at math" can be transmitted from one person to another, and from one generation to the next. Its soci[al] acceptance has come about only because the disease is so common.

Caricatures of Math

My students taught me another interesting lesson: While they professed fear and loathing [of] mathematics, they didn't really know what math is all about. Most of their fears were directed at [a] caricature of mathematics, though admittedly one that is often reinforced in schools.

The students saw mathematics as little more than a bunch of numbers and equations, with no roo[m] for creativity. Moreover, they assumed that mathematics had virtually no relevance to their lives, sin[ce] they didn't plan to be scientists or engineers. It's worth a moment to consider the flaws in thes[e] caricatures.

Numbers and equations are certainly important to mathematics, but they are no more the essence [of] mathematics than paints and paintbrushes are the essence of art. You can see its true essence by lookin[g] to the origin of the word *mathematics* itself, which derives from a Greek term meaning "inclined [to] learn." In other words, mathematics is simply a way of learning about the world around us. It s[o] happens that numbers and equations are very useful to this effort, but we should be careful not [to] confuse the tools with the outcomes.

Once we see that mathematics is a way of learning about the world, it should be immediately cle[ar] that it is a highly creative effort, and that while equations may offer exact solutions, the same may not b[e] true of the mathematical essence. Consider this example: Suppose you deposit $100 into a bank accou[nt] that offers a simple annual interest rate of 3%. How much will you have at the end of one year?

Because 3% of $100 is $3, the "obvious" answer is that you'll have $103 at the end of a year. This [is] probably also the answer that would have gotten full credit in your past math classes. But, of course, i[t's] only true if a whole range of unstated assumptions holds. For example, you have to assume that t[he] bank doesn't fail and doesn't change its interest rate, and that you don't find yourself in need of th[e] money for early withdrawal. In the real world, these assumptions are the parts that require far mo[re] thought and study—more *real* mathematics—than the simple percentage calculation.

As to my students' assumption that mathematics had no relevance to their lives, our housing bubb[le] example should already show that this is far from the truth. Today, mathematics is crucial to almo[st] everything we do. We are regularly faced with financial choices that can make anyone's head spin; ju[st] consider the multitude of cell phone plans you have to select from, the many options you have f[or] education and retirement savings, and the implications of how you deal with medical insurance for bo[th] your bank account and your health. Looking beyond finance, we are confronted almost daily wi[th] decisions that we can make thoughtfully only if we understand basic principles of statistics, which [is] another important part of mathematics. For example, your personal decision on whether to use a c[ell] phone while driving should surely be informed by the statistical research into its dangers, and hardly [a] day goes by without someone telling you why you need this or that to make you healthier or happier[—] claims that you ought to be able to evaluate based on the quality of the statistical evidence backing the[m] up.

The issues go even deeper when we look at the choices we face as voting citizens. We're constantl[y] bombarded by competing claims about the impacts of proposed tax policies or government program[s;] how can you vote intelligently if you don't understand the nature of the economic models used to mak[e] those claims, or if you don't really understand the true meaning of billions and trillions of dollars? An[d] take the issue of global warming: On one side, you're told that it is an issue upon which our very surviv[al] may depend, and on the other side that it is an elaborate hoax. Given that global warming is studied [by] researchers almost entirely through statistical data and mathematical models, how can you decide who[m] to believe if you don't have some understanding of those mathematical ideas yourself?

Getting Good at Math

If you have suffered in the past from fear or loathing of mathematics, then I may be making yo[u] nervous. Although you may now accept that mathematics is important to your life, a book about ma[th] can still seem scary. But it shouldn't. A simple analogy should help.

Just as you don't have to be the Beatles to understand their music, you don't have to be [a] mathematician to understand the way mathematics affects our lives. That is why you won't see a lot [of] equations in this book: The equations in mathematics are like the notes in music. If you want to be [a] songwriter, you'll need to learn the notes, and if you want to be a mathematician (or a scientist [or] engineer or economist), you'll need to learn the equations. But for the kinds of mathematics that we a[ll] encounter every day—the "math for life" that we'll discuss in this book—all you need are those thin[gs] that we talked about before: an open mind and a willingness to learn to think in new ways.

In fact, I'll go so far as to make you the same promise that I've made to my students in the past. ~~you read the whole book, and think carefully as you do so, I promise that you'll find not only that yo~~ *can* understand the mathematics contained here, but that you'll find the topics both useful and fun.

I have just one favor to ask in return: Help in the cause of battling an infectious disease that has bee crippling our society by promising that you'll never again take pride in being "bad at math," and tha you'll do what you can to help others realize that being bad at math should be considered no less a fla than being bad at reading, writing, or thinking.

Crucial Ideas You Didn't Learn in School

Before we delve into all the fun parts, there's one more bit of background we should discuss: why yo haven't learned all this stuff previously.

Consider again the housing bubble example. It is clearly mathematical; its analysis requires a varie of different mathematical concepts, including ratios, percentages, mortgages (which use wh mathematicians call *exponential functions),* statistics, and graphing. Its practical nature is also clear, sin it affected people's lives all over the world. But now ask yourself: Where in the standard mathematic curriculum do we teach students how to deal with such issues?

With rare exceptions (such as college courses in quantitative reasoning), the answer is *nowhere.* Th standard mathematics curriculum begins in grade school with basic arithmetic, then moves on in midd and high school to courses in algebra, geometry, and pre-calculus or calculus. In college, you're either calculus (or beyond) or taking "college versions" of the courses that you didn't fully absorb in hig school, such as college algebra. This standard curriculum covers the crucial mathematical skills neede for students who aspire to careers in science, engineering, economics, or other disciplines that requi advanced mathematical computation. But it almost completely neglects the kind of mathematics th would be most useful to everyone else, including most of the mathematics that arose in our housi bubble study. Notice, for example, that statistics is not part of the standard curriculum, which mea students are not generally taught how to interpret the types of data we discussed in the housing bubb case, or how to analyze graphs like that in Figure 1. And while standard courses may cover exponenti functions and the calculations that underlie mortgage payments, they rarely spend any time examini the implications of those payments, or the factors that should go into deciding whether the payments a affordable.

In other words, despite its clear importance, our schools have by and large neglected to teach "ma for life." But why? The full answer is fairly complex, but the gist of it lies in the perceived purpose teaching mathematics. In decades past, mathematics was seen almost exclusively as a tool for science a engineering, so the curriculum was developed with the goal of putting more people on the science a engineering track. This is a good goal (and one that I strongly support), because there's no question tha we need many more scientists and engineers. It's also an important goal for a society that strives f equality, because studies show that many of the best-paying and most satisfying jobs are ones th require proficiency with tools like those of algebra and calculus. For this reason, I personally believe th we owe it to children to help keep all their options open while they are under our guidance, and therefore think that everyone should be required to learn algebra in high school, and ideally to lea calculus as well. (For anyone who doubts that this is possible, I urge you to watch the movie *Stand a Deliver.)*

But as I have already pointed out, this algebra-track learning is no longer enough. The comple[x] decisions we face today require much greater sophistication with mathematical ideas than was the case [in] the past, and even people who got As in algebra and calculus may not be prepared to evaluate the typ[es] of issues that we'll discuss in this book. I'm far from alone in pointing out this need for greater emphas[is] on quantitative reasoning; many professional societies, including the Mathematical Association [of] America, have produced reports urging such emphasis. Unfortunately, this type of educational chang[e] takes time, and for the most part our high schools and colleges have not yet come around to teaching t[he] kinds of ideas you'll find in this book. In writing it, one of my greatest hopes is that I might make a sm[all] contribution to pushing the needed changes along.

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1. Careful readers may recognize that the math loathers wouldn't necessarily say that they are "bad at math," since they had done well at [it]. However, remember that I was surveying attitudes of college students. Because the math loathers tend to stay just as far away from math [as] the math phobics, over time they tend to forget the mathematics that they once learned, and then become fearful of confronting it again.

Thinking with Numbers

A billion here, a billion there; pretty soon you're talking real money.
— **Attributed to Senator Everett Dirksen**

And now for some temperatures around the nation: 58, 72, 85, 49, 77.
— **George Carlin, comedian**

Question: The following statement appeared in a front-page article in the *New York Times:* "[The percentage of smokers among] eighth graders is up 44 percent, to 10.4 percent." What can you conclude from this statement?

Answer choices:

a. The last time this was studied, the percentage of smokers among eighth graders was negative.

b. The last time this was studied, the percentage of smokers among eighth graders was 10.4%, but now it is 44%.

c. The last time this was studied, the percentage of smokers among eighth graders was about 7.2%.

d. There must be a typo, and the first number should have been 4.4, not 44.

e. The author does not understand percentages, because what is written is impossible.

Before I tell you the correct answer, let me tell you a story that I heard a few years ago at a meeting college mathematics teaching. A group of mathematics faculty had gone to their dean to seek approv for a new course in quantitative reasoning. To explain what the course would cover, they showed him copy of the textbook they hoped to use (of which I am the lead author). The dean scanned the table contents, saw that it has a section on uses and abuses of percentages, and immediately said that th could not teach the course because "percentages are remedial, and we don't give college credit fo remedial courses." The faculty then turned to the page that contained the quote from the above multipl choice question and asked the dean to interpret it. Stumped, he soon acceded to the faculty's request fo the new course.

The lesson here is that being able to compute numbers is not the same thing as being able to *thir* with them. By fifth or sixth grade, most kids have been taught that *percent* means "divided by 100," s we'd certainly expect college students to know that 44% is the same as 44/100, or that 10.4% is the sam as 0.104. But interpreting a statement like "up 44 percent, to 10.4 percent" requires thinking at a muc higher level. You not only need to understand the meaning of the individual percentages, you also nee to think about how they link together. In this case, we're looking for a number that, if you increase it b 44%, ends up at 10.4%. The correct answer is therefore C, because 10.4% is 44% higher than 7.2%. (Yo can check this answer as follows: If you start from 7.2%, then an increase of 44% means an increase 0.44 x 7.2%, which is approximately 3.2%. Adding 3.2% to the starting value of 7.2% gives you th

Statements like "up 44 percent, to 10.4 percent" appear often in news reports, and once yo[u] understand them, you can see that they are a perfectly reasonable way of conveying information. But [as] the college dean story shows, even many well-educated people were never taught how to interpret the[m.] There are at least two reasons why standard curricula do not cover such skills. First, they don't fit in we[ll] with the traditional progression of mathematics. The idea that 44% is 44/100 is nothing more tha[n] division, and therefore can be taught to students in elementary school. In contrast, the interpretation [of] "up 44 percent, to 10.4 percent" requires an implicit understanding of algebra (because finding th[e] starting point of 7.2% involves the process of solving for an unknown variable), along with abstra[ct] reasoning skills that most students don't acquire until at least high school.

A second reason that these skills are rarely taught is that they are more difficult to teach. F[or] example, while "percent" *always* means "divided by 100," the percentage statements in news reports a[re] varied and complex, and sometimes not even stated correctly. There is no single formula that will alway[s] work for interpreting such statements, so we generally learn to deal with them through practice an[d] experience.

In the rest of this chapter, I'll present examples designed to give you some experience at thinking wi[th] the kinds of numbers we see regularly in the news. They should be fun in and of themselves, but I'[ve] chosen them primarily to help you build a basic skill set for quantitative reasoning that we'll then be ab[le] to use in later chapters, in which we'll focus our attention on some of the major issues of our time.

Thinking Big

Most everyone knows that ten is ten times as much as one, that one hundred is ten times as much as te[n,] and that one thousand is ten times as much as one hundred. Knowing those, the meanings of "te[n] thousand" and "one hundred thousand" are fairly obvious. But beyond that, relatively few people reali[ze] that you have to multiply by *one thousand* to make each jump from million to billion to trillion, an[d] even fewer have an intuitive understanding of what these jumps really mean. I don't think it's a[n] exaggeration to say that, for most people, the differences between million, billion, and trillion a[re] primarily in their first letters. Given how often we hear such numbers in the news, it's clearly importan[t] to build better intuition for large numbers. Let's do that by discussing a few simple examples.

Million-dollar athlete. Imagine that you are an elite athlete and sign a contract that pays you $1 milli[on] per year. How long would it take you to earn your first *billion* dollars? If you remember that a billion [is] the same thing as a thousand million, then the answer is obvious: It would take one thousand years [to] earn $1 billion at a rate of $1 million per year. But obvious as the numbers may be, it takes some thoug[ht] to get this result to sink in. A salary of $1 million per year would strike most people as almo[st] unimaginable riches, yet it would take a *thousand years* of such a salary to earn your first billion—whi[ch] still wouldn't put you on the Forbes 400 list of the world's richest people.

Hundred-million-dollar CEO. Now assume you're on the board of directors of a large corporation, an[d] there's a proposal on the table to offer the CEO a pay package worth some $100 million per year (a[n] amount that is high but not unheard of during recent years). The company is profitable and the CEO [is] a smart guy, so you're thinking you'll vote in favor. But then you wonder: Are there other ways th[at]

company could spend the same money that might produce greater long-term value for shareholders? I[...] a subjective question, of course, but here's a thought: Typical salaries for research scientists (with PhD[...] in subjects such as physics, chemistry, and biology) are around $100,000 per year. Let's suppose you[...] company is willing to pay on the high end *and* to add another $100,000 for lab equipment and oth[...] research expenses. Then you'd need $200,000 for each scientist you hire, which means that the $10[...] million that you were going to pay to the CEO could alternatively be used to hire 500 research scientis[...] (because $100 million ÷ $200,000 = 500). I know that some CEOs are very talented people, but you'[...] going to have a hard time convincing me that any one person could produce the same long-term value [...] your company that you'd get from having 500 additional scientists working full-time to help you[...] company come up with new inventions and products. And if you really want to think long term, le[...] allow each of those 500 scientists to take one day a week to go help out with science teaching at a loc[...] school. If we assume that each scientist spends the day with a group of 30 kids, that's 15,000 studen[...] who will be touched by these weekly visits. Aside from the general good that would come of this, don[...] forget that all of them are potential future customers—or future employees—who may remember th[...] *your* company provided the opportunity.

A billion here, a billion there. Now let's move into the realm of the "real money" alluded to in th[...] famous aphorism that opens this chapter. The same math that shows that $100 million could hire 50[...] scientists means that $1 billion could hire 5,000 of them. Going a step further, the $23 billion th[...] Goldman Sachs initially set aside for its bonus pool in one recent year would allow the hiring of mo[...] than 100,000 scientists. Even if you change the assumption from $200,000 to $2 million per scienti[...] thereby allowing plenty of money for building construction, staff expenses, and higher salaries, yo[...] could still hire more than 10,000 scientists. In other words, if the $23 billion were sustainable year aft[...] year, "Goldman Scientific" could become the largest single research institution in the world, with [...] annual operating budget roughly ten times that of major research institutions such as MIT or th[...] University of Texas at Austin. Since I'm a fan of human space exploration, I'll also point out that $2[...] billion is about 25% larger than NASA's budget (roughly $18 billion in 2013), which means it [...] somewhat more than a presidential commission said would have been needed to keep NASA's cancelle[...] "return to the Moon" program on track. So it seems to me that Goldman missed an opportunity to be [...] the forefront of future business opportunities in space, opportunities likely to offer far more long-ter[...] benefit for shareholders than lavishing large paychecks on wizards of finance.

Government money. Even Goldman pales in comparison to the sums that we regularly hear about wi[...] government programs. The biggest sum that's regularly in the news is the federal debt, for which yo[...] might want to calculate your share. If you divide the roughly $17 trillion debt (late 2013) by the roug[...] 315 million people in the United States, you'll find that each person's share of the debt is more tha[...] $50,000, which means that an average family of four owes more than $200,000 to future generations[...] significantly more than it owes for its home. And at the risk of really depressing you, I'll remind you th[...] the debt is not only a burden on the future, but also a burden today because the government must p[...] interest on it. In 2012, for example, the interest totaled $360 billion[2]—which is *more than the total spe[...] by the federal government on education, transportation, and scientific research combined.* Worse, th[...] only reason the interest payment was so "low" was because of record low interest rates. If interest rat[...] rise back up to something more like their average for recent years, the annual interest payments on th[...] current debt could easily double or triple, and that's before we even consider the fact that the debt is st[...] rising. Perhaps, as some politicians argue, we've had no choice but to borrow (and continue to borrow[...]

so much money. But when you consider what else we might do with the money going to interest alon~~it sure makes you think that there ought to be a better way.~~

Counting stars. Let's turn to some big numbers that are less depressing and more amazing. One of m favorites is the number of stars. As you probably know, our Sun is just one of a great many stars tha together, make up what we call our Milky Way Galaxy. The galaxy is so big that no one knows its exac number of stars, but estimates put the number at a few hundred billion. To make the arithmetic easie let's just call it "more than 100 billion." Now, suppose that you're having trouble going to sleep tonight so you decide to count stars. How long would it take you to count 100 billion of them? If we assum that you can count at a rate of one per second, then it would take 100 billion seconds.[3] You can the divide by 60 to convert the 100 billion seconds to minutes, divide by 60 again to convert it to hour divide by 24 to convert it to days, and divide by 365 to convert it to years. Try it on your calculator, an you'll find that 100 billion seconds is almost 3,200 years. In other words, it would take more than 3,00 years just to *count* 100 billion stars in our galaxy, assuming that you never take a break, never go sleep, and manage to stay alive for a few thousand years. And that's just the stars in *our* galaxy.

If you multiply the 100 billion stars in a typical galaxy by the estimated 100 billion galaxies in t known universe, you'll find that the total number of stars in our universe is abo 10,000,000,000,000,000,000,000 (a 1 followed by 22 zeros, or 10^{22}), which you could say as "10 billi trillion," or "10 million quadrillion," or "10,000 billion billion." But rather than giving it a name, I prefe a more interesting comparison. You can estimate the number of grains of sand in a box by dividing t volume of the box (which is its length times its width times its depth) by the average volume of a sing sand grain. In the same basic way, you can estimate the number of grains of sand on all the beaches Earth by finding the total volume of beach sand and dividing by the average volume of sand grain Estimating the total volume of beach sand on Earth is not as difficult as it sounds, though like mo measurements, it's much easier if you use metric units. A quick Web search will tell you that the tot length of sandy beach on Earth is about 360,000 kilometers (about 220,000 miles), and the avera beach is about 50 meters wide and 4 meters deep. I'll leave the rest of the multiplication (and division the average sand grain volume) to interested readers, and just tell you the amazing result: *the number grains of sand on all the beaches on Earth is comparable to the number of stars in the known univer* Next time you're thinking about whether there might be other civilizations out there, remember that comparison to all the stars in the universe, our Sun is like just one grain of sand among all the grains all the beaches on Earth combined.

Until the Sun dies. As the examples of star counting show, astronomy is a subject full of amazemen and one that should make us proud to be members of a species that has managed to learn su incredible things about our universe. But astronomy sometimes seems scary, too, especially when yo learn, for example, that the Sun is doomed to die. Fortunately, a little math should relieve any concer you might have. The Sun is indeed doomed to die, but not for about 5 billion years. How long is billion years? One way to put it in perspective is to compare it to a human lifetime. If we assume lifetime of 100 years, then 5 billion years is about 50 million lifetimes. It turns out that 100 years als happens to be close to 50 million minutes (which you can see by taking 100 years and multiplying 365 days in a year, 24 hours per day, and 60 minutes per hour). We can therefore say that a huma lifetime compared to the remaining life of the Sun is like a mere minute in a long human life. Huma creations register only a little more on the Sun's time scale. The Egyptian pyramids have often bee described as "eternal," but at their current rates of erosion, they will have turned to dust within abou

500,000 years. That may sound like a long time, but the Sun's remaining lifetime is some 10,000 time~~longer. Clearly, we have more pressing things to worry about than the eventual death of our Sun.~~

Another way to consider the Sun's remaining 5 billion years is to think about what would happen we ended up doing ourselves in. No matter how much damage we do to our planet, we won't wipe ou life entirely. If we cause our own extinction, it's likely that some other species will eventually evolv intelligence as great as ours, giving Earth another chance to have a civilization that makes good rath than destroying itself. There's no way to know exactly how long it would take for the next intelligence t arise, but I'd say that 50 million years is a pretty conservative guess. In that case, if the intelligent being that rise up 50 million years from now also wipe themselves out, another intelligence could presumab emerge some 50 million years after that, and so on. You might think that at 50 million years per sho Earth would quickly run out of opportunities. But it wouldn't: The 5 billion years remaining in the Sun lifetime would be enough for Earth to have 100 more chances for an intelligent species to rise up, each 5 million years after the last one. It's truly incredible to think about, and it makes you think tha eventually, there would be a species smart enough to travel to the stars and thereby eliminate wor about what happens when the Sun dies. We can only hope that species will be us.

Lunch with your students. Having talked about numbers in the millions, billions, and trillions, it's eas to start thinking that anything in the thousands must be small. But even those numbers are much larg than we usually recognize. Imagine that a university with 25,000 students (typical of many sta universities) hires a new president. Thinking that he should get to know the students, the presiden offers to meet for lunch with groups of 5 students at a time. If all 25,000 students accept, how long w it take the president to finish all the lunches? Again, the basic math is straightforward. If he holds t lunches 5 days a week, with 5 students at a time, then he'll be dining with 25 students per week. If leave 2 weeks off for the winter holidays and 10 weeks for summer, he could have these lunches 4 weeks per year, which means the lunches would include a total of 40 x 25 = 1,000 students each year. A that rate, it would take him 25 years to get through the lunches with all 25,000 students—but, of cours that wouldn't work, since most of the students would have graduated long before getting their turn.

Incidentally, similar thinking probably explains why "special interests" have become so dominant i politics. The U.S. House of Representatives has 435 members; dividing this number into the U.S population of about 315 million people, we find that each representative has an average of more tha 700,000 constituents. If we assume a 40-hour workweek, 50 weeks per year, then each representative h about 2,000 working hours per year, or 4,000 hours during a two-year term. If you divide that by t 700,000 constituents, you'll find that a representative could at best devote about 20 seconds to ea constituent (on average). Given this reality, along with the reality that it can take millions of dollars run a campaign, it's no wonder that the representatives devote most of their listening time to th relatively small numbers of people who fund the bulk of their campaigns.

Stadium lottery. A different type of big-number thinking requires putting various odds into perspectiv As an example, imagine watching a football game in a stadium filled to capacity with 50,000 people. Th announcer comes on and says that if everyone is willing to ante up $500 each, the league will pick o person at random to receive a multimillion-dollar prize. Would you pay the $500? Probably not; aft all, when you look around at a stadium full of people, it seems almost impossible to believe that you'd b the one person selected at random, and it certainly wouldn't seem worth spending $500 for that tir chance. Yet outside the stadium, nearly half of all Americans play this very game every year. That because people who play the lottery (which about half of all Americans do) spend an average of abo

$500 per year on their lottery tickets, while each person's chance of being a big winner is no bigger tha~~n~~ the ~~chance of being that one person selected in the stadium. In fact, it's actually smaller, since the tren~~d has been for lotteries to offer larger prizes with worse odds. To put it a different way, even if you spen~~d~~ $500 per year—which adds up to $20,000 over a 40-year playing "career"—the chance that you'll ever ~~b~~e one of the big winners is only about 1 in 50,000. So to all the lottery players out there, consider th~~is~~ statement of fact: While *someone* will surely win, I can be 99.998% certain that it won't ever be you.[4] St~~ill want to play, or can you think of better uses for your $20,000? As a widely circulated Internet messag~~e~~ says, the lottery is essentially "a tax on people who are bad at math."

The same basic ideas apply to gambling of all types. When you walk into a casino, the odds hav~~e~~ been stacked against you—that's why the casino has money to offer you all those free drinks and othe~~r~~ enticements. If you think of yourself in the stadium full of people, you'll probably realize how crazy it ~~is~~ to start gambling. But when it's just you and the machine, or you and the card dealer, it can sudden~~ly~~ seem like you must be bound to win. Moreover, the gambling companies have spent hundreds ~~of~~ millions of dollars on research to find the best ways to convince you to keep playing, with lighting, bel~~ls~~ and other tricks of the trade designed to make you think you have more of a chance than you really d~~o. Frankly, I think this gives the casinos a fundamentally unfair advantage over their patrons, and if it we~~re~~ up to me I'd require all casinos to post large warning labels, much like those we require on cigarettes. I~~n this case, they could read something like: "WARNING: The games in this facility are set up so that th~~e~~ odds are stacked against you. While an individual may occasionally come out ahead after any particul~~ar~~ play, continued playing virtually guarantees that you will lose money in the end."

Dealing with Uncertainty

In math classes, you were probably told to assume that the numbers you dealt with were always exact. I~~n science classes, you may have learned that measurements have associated uncertainties, and learne~~d~~ techniques for dealing with those uncertainties. The situation is more difficult in the real world, whe~~re we may not even have a good way to estimate the uncertainty associated with the numbers w~~e~~ encounter.

Consider the forecasts we hear each year about future budget deficits. In 2008, for example, th~~e~~ president's budget office predicted that the deficit for 2009 would be $187.166 billion. Notice that th~~e~~ number was stated to the nearest $0.001 billion, which is the same as the nearest $1 million. When 200~~9 ended, the actual deficit turned out to be $1.42 *trillion*—which means that although the defic~~it prediction had been stated as though we knew it to the nearest million dollars, in reality we didn't eve~~n know it to the nearest *trillion* dollars!

In fairness, the budget office is staffed by pretty smart people, and they were well aware that the~~y couldn't really know the future deficit to the nearest million dollars. Their full report included hundred~~s of pages that outlined various assumptions that would have had to be true for the numbers to come o~~ut exactly as predicted, along with descriptions of various uncertainties that could also affect th~~e predictions. However, when budget numbers appear in the news media, all those caveats usuall~~y disappear, which can mislead you into thinking that the numbers are known far better than they real~~ly are.

The fact that numbers are so often reported without clear descriptions of their uncertainties mean~~s we must develop ways of looking critically at all the numbers we encounter. Rather than proceedi~~ng

through specific examples as we did with big numbers, I'll suggest four general ways of thinking abo
uncertainties.

Accuracy versus precision. Although many people interchange the words *accuracy* and *precision,* th
are not quite the same thing. To understand the distinction, imagine that you actually weigh 125
pounds, and that you check your weight on two different scales. One scale is the old-fashioned type th
you can at best read to about the nearest pound, and it says you weigh 125 pounds. The other scale
digital, and it says you weigh 121.44 pounds. We say that the reading on the digital scale is "precise t
the nearest 0.01 pound," while the reading on the old-fashioned scale is "precise to the nearest pound
This means the digital scale is more precise. However, because the old-fashioned scale got closer to you
actual weight, it is more accurate. In other words, accuracy describes how closely the measureme
approximates the true value, while precision describes the amount of detail in the measurement.

You can probably see how unwarranted precision can cause problems. For example, stating a weigl
as 125 pounds implies that you know it to the nearest pound, while stating a weight as 121.44 poun
implies that you know it to the nearest 0.01 pound. In this case, the fact that your actual weight w
125.2 pounds means the first statement was true (a weight of 125 really is correct to the nearest poun
while the second statement was false. More generally, stating a number with more precision than
justified is always deceptive, because it implies that you know more than you really do.

Let's apply this idea to the budget deficit example. When the 2009 deficit projection was stated to th
nearest $1 million, it implied that it was accurate within this amount. Given that the projection turne
out to be wrong by more than $1 trillion, and that a trillion is a million times a million, the actu
uncertainty in the budget estimate was *a million times worse* than the implied uncertainty of $1 millio
We can't really blame the budget office, since they had those hundreds of pages that explained all th
caveats. The blame, if any, should go to the media that reported the number as though we really d
know it that well.

The 2010 census provides another good example. According to the published reports, the censu
found that the U.S. population on April 1, 2010, was 308,745,538. But there's no way that anyon
could really know the population exactly. Aside from the inevitable difficulties of counting, the fact th
an average of about eight births and four deaths occur *each minute* in the United States means that yo
could only know the exact population if there were some way to count everyone instantaneously, whi
the census was carried out over a period of many months. Like the budget office, the Census Bureau w
well aware that the number was not really known as well as its precision implied. In fact, if you read t
full census report, you'll find that the Census Bureau estimated the uncertainty in the population cou
to be at least three million people, meaning the actual population could easily have been three milli
higher or lower than the reported value.

The bottom line is that many of the numbers that we hear in the news are reported with mo
precision than they deserve, falsely implying a level of accuracy that doesn't really exist. So the first lesso
in dealing with uncertainty in the news is to beware of any number you hear, and to think carefull
about whether it can really be as precise as reported. Given the news media's propensity to leave out a
the important caveats, when possible you should go back to original sources (such as the budg
documents or Census Bureau reports) to find out what has been ignored.

Random versus systematic errors. Numbers may be inaccurate for a variety of different reasons, but
most cases we can divide those reasons into two broad classes: *random errors* that occur because
unpredictable events in the measurement process, and *systematic errors* that result from some problem

the way the measurement system is designed.

Consider the potential sources of inaccuracy in the census count of the U.S. population. Some erro[rs] may occur because people fill out the census surveys incorrectly, or because census workers ma[ke] mistakes when they enter the survey data into their computers. These types of accidental errors a[re] random errors, because we cannot predict whether any individual error overcounts or undercounts t[he] population. In contrast, consider errors that occur because census workers can't find all the homeless [or] all of the very poor, or because undocumented aliens try to hide their presence. These are systemat[ic] errors that arise because the system is unable to account for all the people in those groups, and the[se] particular systematic errors can only lead to an undercount. Other types of systematic errors can lead [to] overcounts; for example, college students may be counted both by their parents and in their housing [at] school, and children of divorced parents may be counted in both households.

Perhaps the most important distinction between random and systematic errors is that while ther[e is] nothing you can do about random errors after they've occurred (though well-designed systems ca[n] minimize the likelihood of their occurrence), you can correct for systematic errors if you are aware [of] them. For example, by looking for the homeless, the poor, or undocumented aliens with extra care in [a] few selected areas, the Census Bureau can estimate the amount by which its standard processes tend [to] undercount these groups. Indeed, the Census Bureau has data available that should in principle allow [it] to make its population estimate more accurate—but it is allowed to use these data only for limit[ed] purposes. Part of the problem revolves around a constitutional question: The U.S. Constitution (Artic[le] 1, Section 2, Subsection 2) calls for an "actual enumeration" of the population. Those who oppose th[e] use of statistical data to improve the population estimate point out that "enumeration" seems to imply [a] one-by-one count. Those who favor using the statistical data point out that an exact count is impossibl[e,] and therefore focus on the word "actual," arguing that statistics can help us get closer to the actual valu[e.] Of course, the real issue is probably more political than constitutional: Democrats tend to favor the u[se] of statistical data because it leads to higher numbers of people who tend to vote Democratic, whi[le] Republicans oppose the use of statistical data for the same reason. Note that this debate is not just abo[ut] voting. The census results affect the makeup of Congress and of state legislatures, because they are use[d] to apportion political representation by state and by locality. The census results also have econom[ic] value, because states and cities receive allotments of federal money based on their populations.

Absolute versus relative errors. There are two basic ways to think about the sizes of errors. First, we ca[n] think about the *absolute error,* meaning the actual amount by which a given number differs from its tru[e] value. Alternatively, we can consider the *relative error,* which describes the size of the error [in] comparison to the true value. A simple example should illustrate the point. If the government ev[er] managed to predict the budget deficit to within about $1 million, we'd be very impressed, because $[1] million is so small compared to the trillions of dollars that the government collects and spends. But [if] your electric company overcharged you by $1 million, the error would seem enormous. In other word[s,] both cases have the same absolute error of $1 million, but the relative error is much smaller for the defic[it] than for your electric bill. By the way, in case you haven't thought it through fully yet, this idea explai[ns] the famous quote from Senator Dirksen: Politicians can throw around dollars like "a billion here, [a] billion there" because billions are relatively small in a federal budget that is measured in trillions, b[ut] there's no doubt that in absolute terms, we're talking "real money."

Measurements versus models. So far we've talked about the interpretation of numbers and the[ir] uncertainties, but it's also important to consider where numbers come from in the first place. F[or]

example, a weight on a scale represents a simple measurement, while a prediction about a future budg deficit represents the result of a complex economic model that may have tens of thousands of variable all evaluated by a computer that performs millions of calculations. Although it's possible that a weigh measurement could have a relative error as large as that of a budget prediction, it's also pretty obviou that the budget prediction has many more ways to go wrong. Economists and scientists test models using them to try to reproduce measurements made in the past. For example, if your economic mod can successfully "predict" last year's deficit from information that was available before the year began then you would have at least some reason to trust its prediction for next year. Of course, unforesee circumstances could still make the model quite wrong, as was the case with the 2009 deficit predictic that we've discussed. Among other problems, the model used in that prediction did not take in account the collapse of the housing market or the massive government bailouts that followed.

Apples and Oranges

The famous saying that you can't add apples and oranges reflects a deeper idea about the numbers w encounter in daily life, which is that numbers are almost always associated with some type, or *unit,* measurement. If you have five apples and three oranges, you can think of the units as apples ar oranges, and because these units are different, you can't combine them.

Units provide crucial context to numbers. If I say that a person weighs 75, the meaning is qui different if I mean pounds than if I mean kilograms. Similarly, a temperature of 32 is pretty hot if you'r in Europe, where temperatures are reported on the Celsius scale, but it's freezing on the Fahrenheit sca used in the United States. Of course, units alone may not provide all the context needed; the Georg Carlin quote at the beginning of the chapter is funny not because he didn't distinguish between Celsiu and Fahrenheit, but because he left out the critical context of locations.

For the most part, news media are pretty good about stating units; you'll rarely hear a numb reported without it being clear whether the number represents dollars, pounds, people, or something els So our reason for discussing units has less to do with the news and more to do with the ways in whic they can help us think about quantitative problems. In fact, *unit analysis* is arguably the simplest ar most useful of all problem-solving techniques—yet it is rarely discussed in math classes (though ofte covered in science classes). To get started with unit analysis, you need only remember two simple idea the word *per* implies division, while *of* implies multiplication.

As an example, imagine that you're trying to figure out the gas mileage you're getting, but aren't sui how to do it. If you remember that gas mileage is usually given in units of miles per gallon, yo immediately recognize that you need to take something with units of miles and divide it by somethin with units of gallons. From there, it's a small step to realize that you should divide the number of mile you've driven since you last filled your gas tank by the number of gallons it takes to fill up. For examp if you drove 200 miles on 8 gallons of gas, then your mileage is 200 miles ÷ 8 gallons = 25 miles p gallon. Similarly, you can always remember that speed is a distance divided by a time just by recallin that we measure highway speeds in "miles per hour."

Cases with *of* are similarly easy. Suppose you buy 10 pounds of apples at a price of $3 per pound. TI word *of* (in "price of $3") tells us to multiply, so the total price is 10 pounds x $3/pound = $30. Notic how the pound units cancel out to leave dollars: This happens because the first number is in pound while the second number divides by pounds, and anything divided by itself is just a plain number one.

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