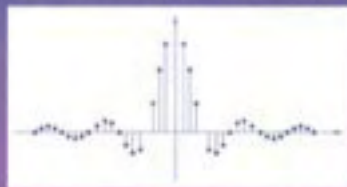
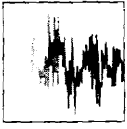


*Third Edition*  
**DIGITAL  
SIGNAL  
PROCESSING**

*Principles, Algorithms, and Applications*



John G. Proakis  
Dimitris G. Manolakis



# Digital Signal Processing

Principles, Algorithms, and Applications

Third Edition

**John G. Proakis**

Northeastern University

**Dimitris G. Manolakis**

Boston College



PRENTICE-HALL INTERNATIONAL, INC.

---

This edition may be sold only in those countries to which it is consigned by Prentice-Hall International. It is not to be reexported and it is not for sale in the U.S.A., Mexico, or Canada.



© 1996 by Prentice-Hall, Inc.  
Simon & Schuster/A Viacom Company  
Upper Saddle River, New Jersey 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

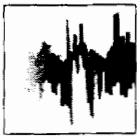
The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Printed in the United States of America

10 9 8 7 6 5

ISBN 0-13-394338-9

Prentice-Hall International (UK) Limited, *London*  
Prentice-Hall of Australia Pty. Limited, *Sydney*  
Prentice-Hall Canada, Inc., *Toronto*  
Prentice-Hall Hispanoamericana, S.A., *Mexico*  
Prentice-Hall of India Private Limited, *New Delhi*  
Prentice-Hall of Japan, Inc., *Tokyo*  
Simon & Schuster Asia Pte. Ltd., *Singapore*  
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*  
Prentice-Hall, Inc. *Upper Saddle River, New Jersey*



# Contents

<b>PREFACE</b>	<b>xiii</b>
<b>1 INTRODUCTION</b>	<b>1</b>
1.1 Signals, Systems, and Signal Processing	2
1.1.1 Basic Elements of a Digital Signal Processing System	4
1.1.2 Advantages of Digital over Analog Signal Processing	5
1.2 Classification of Signals	6
1.2.1 Multichannel and Multidimensional Signals	7
1.2.2 Continuous-Time Versus Discrete-Time Signals	8
1.2.3 Continuous-Valued Versus Discrete-Valued Signals	10
1.2.4 Deterministic Versus Random Signals	11
1.3 The Concept of Frequency in Continuous-Time and Discrete-Time Signals	14
1.3.1 Continuous-Time Sinusoidal Signals	14
1.3.2 Discrete-Time Sinusoidal Signals	16
1.3.3 Harmonically Related Complex Exponentials	19
1.4 Analog-to-Digital and Digital-to-Analog Conversion	21
1.4.1 Sampling of Analog Signals	23
1.4.2 The Sampling Theorem	29
1.4.3 Quantization of Continuous-Amplitude Signals	33
1.4.4 Quantization of Sinusoidal Signals	36
1.4.5 Coding of Quantized Samples	38
1.4.6 Digital-to-Analog Conversion	38
1.4.7 Analysis of Digital Signals and Systems Versus Discrete-Time Signals and Systems	39
1.5 Summary and References	39
Problems	40

**2 DISCRETE-TIME SIGNALS AND SYSTEMS****43**

- 2.1 Discrete-Time Signals 43
  - 2.1.1 Some Elementary Discrete-Time Signals, 45
  - 2.1.2 Classification of Discrete-Time Signals, 47
  - 2.1.3 Simple Manipulations of Discrete-Time Signals, 52
- 2.2 Discrete-Time Systems 56
  - 2.2.1 Input–Output Description of Systems, 56
  - 2.2.2 Block Diagram Representation of Discrete-Time Systems, 59
  - 2.2.3 Classification of Discrete-Time Systems, 62
  - 2.2.4 Interconnection of Discrete-Time Systems, 70
- 2.3 Analysis of Discrete-Time Linear Time-Invariant Systems 72
  - 2.3.1 Techniques for the Analysis of Linear Systems, 72
  - 2.3.2 Resolution of a Discrete-Time Signal into Impulses, 74
  - 2.3.3 Response of LTI Systems to Arbitrary Inputs: The Convolution Sum, 75
  - 2.3.4 Properties of Convolution and the Interconnection of LTI Systems, 82
  - 2.3.5 Causal Linear Time-Invariant Systems, 86
  - 2.3.6 Stability of Linear Time-Invariant Systems, 87
  - 2.3.7 Systems with Finite-Duration and Infinite-Duration Impulse Response, 90
- 2.4 Discrete-Time Systems Described by Difference Equations 91
  - 2.4.1 Recursive and Nonrecursive Discrete-Time Systems, 92
  - 2.4.2 Linear Time-Invariant Systems Characterized by Constant-Coefficient Difference Equations, 95
  - 2.4.3 Solution of Linear Constant-Coefficient Difference Equations, 100
  - 2.4.4 The Impulse Response of a Linear Time-Invariant Recursive System, 108
- 2.5 Implementation of Discrete-Time Systems 111
  - 2.5.1 Structures for the Realization of Linear Time-Invariant Systems, 111
  - 2.5.2 Recursive and Nonrecursive Realizations of FIR Systems, 116
- 2.6 Correlation of Discrete-Time Signals 118
  - 2.6.1 Crosscorrelation and Autocorrelation Sequences, 120
  - 2.6.2 Properties of the Autocorrelation and Crosscorrelation Sequences, 122
  - 2.6.3 Correlation of Periodic Sequences, 124
  - 2.6.4 Computation of Correlation Sequences, 130
  - 2.6.5 Input–Output Correlation Sequences, 131
- 2.7 Summary and References 134
- Problems 135

---

<b>3</b>	<b>THE Z-TRANSFORM AND ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEMS</b>	<b>151</b>
3.1	The $z$ -Transform 151	
3.1.1	The Direct $z$ -Transform. 152	
3.1.2	The Inverse $z$ -Transform. 160	
3.2	Properties of the $z$ -Transform 161	
3.3	Rational $z$ -Transforms 172	
3.3.1	Poles and Zeros. 172	
3.3.2	Pole Location and Time-Domain Behavior for Causal Signals. 178	
3.3.3	The System Function of a Linear Time-Invariant System. 181	
3.4	Inversion of the $z$ -Transform 184	
3.4.1	The Inverse $z$ -Transform by Contour Integration. 184	
3.4.2	The Inverse $z$ -Transform by Power Series Expansion. 186	
3.4.3	The Inverse $z$ -Transform by Partial-Fraction Expansion. 188	
3.4.4	Decomposition of Rational $z$ -Transforms. 195	
3.5	The One-sided $z$ -Transform 197	
3.5.1	Definition and Properties. 197	
3.5.2	Solution of Difference Equations. 201	
3.6	Analysis of Linear Time-Invariant Systems in the $z$ -Domain 203	
3.6.1	Response of Systems with Rational System Functions. 203	
3.6.2	Response of Pole-Zero Systems with Nonzero Initial Conditions. 204	
3.6.3	Transient and Steady-State Responses. 206	
3.6.4	Causality and Stability. 208	
3.6.5	Pole-Zero Cancellations. 210	
3.6.6	Multiple-Order Poles and Stability. 211	
3.6.7	The Schür-Cohn Stability Test. 213	
3.6.8	Stability of Second-Order Systems. 215	
3.7	Summary and References 219	
	Problems 220	
<b>4</b>	<b>FREQUENCY ANALYSIS OF SIGNALS AND SYSTEMS</b>	<b>230</b>
4.1	Frequency Analysis of Continuous-Time Signals 230	
4.1.1	The Fourier Series for Continuous-Time Periodic Signals. 232	
4.1.2	Power Density Spectrum of Periodic Signals. 235	
4.1.3	The Fourier Transform for Continuous-Time Aperiodic Signals. 240	
4.1.4	Energy Density Spectrum of Aperiodic Signals. 243	
4.2	Frequency Analysis of Discrete-Time Signals 247	
4.2.1	The Fourier Series for Discrete-Time Periodic Signals. 247	

- 4.2.2 Power Density Spectrum of Periodic Signals. 250
- 4.2.3 The Fourier Transform of Discrete-Time Aperiodic Signals. 253
- 4.2.4 Convergence of the Fourier Transform. 256
- 4.2.5 Energy Density Spectrum of Aperiodic Signals. 260
- 4.2.6 Relationship of the Fourier Transform to the  $z$ -Transform. 264
- 4.2.7 The Cepstrum. 265
- 4.2.8 The Fourier Transform of Signals with Poles on the Unit Circle. 267
- 4.2.9 The Sampling Theorem Revisited. 269
- 4.2.10 Frequency-Domain Classification of Signals: The Concept of Bandwidth. 279
- 4.2.11 The Frequency Ranges of Some Natural Signals. 282
- 4.2.12 Physical and Mathematical Dualities. 282
- 4.3 Properties of the Fourier Transform for Discrete-Time Signals 286
  - 4.3.1 Symmetry Properties of the Fourier Transform. 287
  - 4.3.2 Fourier Transform Theorems and Properties. 294
- 4.4 Frequency-Domain Characteristics of Linear Time-Invariant Systems 305
  - 4.4.1 Response to Complex Exponential and Sinusoidal Signals: The Frequency Response Function. 306
  - 4.4.2 Steady-State and Transient Response to Sinusoidal Input Signals. 314
  - 4.4.3 Steady-State Response to Periodic Input Signals. 315
  - 4.4.4 Response to Aperiodic Input Signals. 316
  - 4.4.5 Relationships Between the System Function and the Frequency Response Function. 319
  - 4.4.6 Computation of the Frequency Response Function. 321
  - 4.4.7 Input-Output Correlation Functions and Spectra. 325
  - 4.4.8 Correlation Functions and Power Spectra for Random Input Signals. 327
- 4.5 Linear Time-Invariant Systems as Frequency-Selective Filters 330
  - 4.5.1 Ideal Filter Characteristics. 331
  - 4.5.2 Lowpass, Highpass, and Bandpass Filters. 333
  - 4.5.3 Digital Resonators. 340
  - 4.5.4 Notch Filters. 343
  - 4.5.5 Comb Filters. 345
  - 4.5.6 All-Pass Filters. 350
  - 4.5.7 Digital Sinusoidal Oscillators. 352
- 4.6 Inverse Systems and Deconvolution 355
  - 4.6.1 Invertibility of Linear Time-Invariant Systems. 356
  - 4.6.2 Minimum-Phase, Maximum-Phase, and Mixed-Phase Systems. 359
  - 4.6.3 System Identification and Deconvolution. 363
  - 4.6.4 Homomorphic Deconvolution. 365

---

4.7	Summary and References	367
	Problems	368
<b>5</b>	<b>THE DISCRETE FOURIER TRANSFORM: ITS PROPERTIES AND APPLICATIONS</b>	<b>394</b>
5.1	Frequency Domain Sampling: The Discrete Fourier Transform	394
5.1.1	Frequency-Domain Sampling and Reconstruction of Discrete-Time Signals.	394
5.1.2	The Discrete Fourier Transform (DFT).	399
5.1.3	The DFT as a Linear Transformation.	403
5.1.4	Relationship of the DFT to Other Transforms.	407
5.2	Properties of the DFT	409
5.2.1	Periodicity, Linearity, and Symmetry Properties.	410
5.2.2	Multiplication of Two DFTs and Circular Convolution.	415
5.2.3	Additional DFT Properties.	421
5.3	Linear Filtering Methods Based on the DFT	425
5.3.1	Use of the DFT in Linear Filtering.	426
5.3.2	Filtering of Long Data Sequences.	430
5.4	Frequency Analysis of Signals Using the DFT	433
5.5	Summary and References	440
	Problems	440
<b>6</b>	<b>EFFICIENT COMPUTATION OF THE DFT: FAST FOURIER TRANSFORM ALGORITHMS</b>	<b>448</b>
6.1	Efficient Computation of the DFT: FFT Algorithms	448
6.1.1	Direct Computation of the DFT.	449
6.1.2	Divide-and-Conquer Approach to Computation of the DFT.	450
6.1.3	Radix-2 FFT Algorithms.	456
6.1.4	Radix-4 FFT Algorithms.	465
6.1.5	Split-Radix FFT Algorithms.	470
6.1.6	Implementation of FFT Algorithms.	473
6.2	Applications of FFT Algorithms	475
6.2.1	Efficient Computation of the DFT of Two Real Sequences.	475
6.2.2	Efficient Computation of the DFT of a $2N$ -Point Real Sequence.	476
6.2.3	Use of the FFT Algorithm in Linear Filtering and Correlation.	477
6.3	A Linear Filtering Approach to Computation of the DFT	479
6.3.1	The Goertzel Algorithm.	480
6.3.2	The Chirp- $z$ Transform Algorithm.	482



- 6.4 Quantization Effects in the Computation of the DFT 486
  - 6.4.1 Quantization Errors in the Direct Computation of the DFT. 487
  - 6.4.2 Quantization Errors in FFT Algorithms. 489
- 6.5 Summary and References 493
  - Problems 494

## **7 IMPLEMENTATION OF DISCRETE-TIME SYSTEMS**

500

- 7.1 Structures for the Realization of Discrete-Time Systems 500
- 7.2 Structures for FIR Systems 502
  - 7.2.1 Direct-Form Structure, 503
  - 7.2.2 Cascade-Form Structures, 504
  - 7.2.3 Frequency-Sampling Structures<sup>†</sup>, 506
  - 7.2.4 Lattice Structure, 511
- 7.3 Structures for IIR Systems 519
  - 7.3.1 Direct-Form Structures, 519
  - 7.3.2 Signal Flow Graphs and Transposed Structures, 521
  - 7.3.3 Cascade-Form Structures, 526
  - 7.3.4 Parallel-Form Structures, 529
  - 7.3.5 Lattice and Lattice-Ladder Structures for IIR Systems, 531
- 7.4 State-Space System Analysis and Structures 539
  - 7.4.1 State-Space Descriptions of Systems Characterized by Difference Equations, 540
  - 7.4.2 Solution of the State-Space Equations, 543
  - 7.4.3 Relationships Between Input-Output and State-Space Descriptions, 545
  - 7.4.4 State-Space Analysis in the  $z$ -Domain, 550
  - 7.4.5 Additional State-Space Structures, 554
- 7.5 Representation of Numbers 556
  - 7.5.1 Fixed-Point Representation of Numbers, 557
  - 7.5.2 Binary Floating-Point Representation of Numbers, 561
  - 7.5.3 Errors Resulting from Rounding and Truncation, 564
- 7.6 Quantization of Filter Coefficients 569
  - 7.6.1 Analysis of Sensitivity to Quantization of Filter Coefficients, 569
  - 7.6.2 Quantization of Coefficients in FIR Filters, 578
- 7.7 Round-Off Effects in Digital Filters 582
  - 7.7.1 Limit-Cycle Oscillations in Recursive Systems, 583
  - 7.7.2 Scaling to Prevent Overflow, 588
  - 7.7.3 Statistical Characterization of Quantization Effects in Fixed-Point Realizations of Digital Filters, 590
- 7.8 Summary and References 598
  - Problems 600

**8 DESIGN OF DIGITAL FILTERS****614**

- 8.1 General Considerations 614
  - 8.1.1 Causality and Its Implications, 615
  - 8.1.2 Characteristics of Practical Frequency-Selective Filters, 619
- 8.2 Design of FIR Filters 620
  - 8.2.1 Symmetric and Antisymmetric FIR Filters, 620
  - 8.2.2 Design of Linear-Phase FIR Filters Using Windows, 623
  - 8.2.3 Design of Linear-Phase FIR Filters by the Frequency-Sampling Method, 630
  - 8.2.4 Design of Optimum Equiripple Linear-Phase FIR Filters, 637
  - 8.2.5 Design of FIR Differentiators, 652
  - 8.2.6 Design of Hilbert Transformers, 657
  - 8.2.7 Comparison of Design Methods for Linear-Phase FIR Filters, 662
- 8.3 Design of IIR Filters From Analog Filters 666
  - 8.3.1 IIR Filter Design by Approximation of Derivatives, 667
  - 8.3.2 IIR Filter Design by Impulse Invariance, 671
  - 8.3.3 IIR Filter Design by the Bilinear Transformation, 676
  - 8.3.4 The Matched- $z$  Transformation, 681
  - 8.3.5 Characteristics of Commonly Used Analog Filters, 681
  - 8.3.6 Some Examples of Digital Filter Designs Based on the Bilinear Transformation, 692
- 8.4 Frequency Transformations 692
  - 8.4.1 Frequency Transformations in the Analog Domain, 693
  - 8.4.2 Frequency Transformations in the Digital Domain, 698
- 8.5 Design of Digital Filters Based on Least-Squares Method 701
  - 8.5.1 Padé Approximation Method, 701
  - 8.5.2 Least-Squares Design Methods, 706
  - 8.5.3 FIR Least-Squares Inverse (Wiener) Filters, 711
  - 8.5.4 Design of IIR Filters in the Frequency Domain, 719
- 8.6 Summary and References 724
  - Problems 726

**9 SAMPLING AND RECONSTRUCTION OF SIGNALS****738**

- 9.1 Sampling of Bandpass Signals 738
  - 9.1.1 Representation of Bandpass Signals, 738
  - 9.1.2 Sampling of Bandpass Signals, 742
  - 9.1.3 Discrete-Time Processing of Continuous-Time Signals, 746
- 9.2 Analog-to-Digital Conversion 748
  - 9.2.1 Sample-and-Hold, 748
  - 9.2.2 Quantization and Coding, 750
  - 9.2.3 Analysis of Quantization Errors, 753
  - 9.2.4 Oversampling A/D Converters, 756

- 9.3 Digital-to-Analog Conversion 763
  - 9.3.1 Sample and Hold, 765
  - 9.3.2 First-Order Hold, 768
  - 9.3.3 Linear Interpolation with Delay, 771
  - 9.3.4 Oversampling D/A Converters, 774
- 9.4 Summary and References 774
  - Problems 775

**10 MULTIRATE DIGITAL SIGNAL PROCESSING****782**

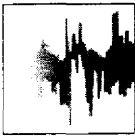
- 10.1 Introduction 783
- 10.2 Decimation by a Factor  $D$  784
- 10.3 Interpolation by a Factor  $I$  787
- 10.4 Sampling Rate Conversion by a Rational Factor  $I/D$  790
- 10.5 Filter Design and Implementation for Sampling-Rate Conversion 792
  - 10.5.1 Direct-Form FIR Filter Structures, 793
  - 10.5.2 Polyphase Filter Structures, 794
  - 10.5.3 Time-Variant Filter Structures, 800
- 10.6 Multistage Implementation of Sampling-Rate Conversion 806
- 10.7 Sampling-Rate Conversion of Bandpass Signals 810
  - 10.7.1 Decimation and Interpolation by Frequency Conversion, 812
  - 10.7.2 Modulation-Free Method for Decimation and Interpolation, 814
- 10.8 Sampling-Rate Conversion by an Arbitrary Factor 815
  - 10.8.1 First-Order Approximation, 816
  - 10.8.2 Second-Order Approximation (Linear Interpolation), 819
- 10.9 Applications of Multirate Signal Processing 821
  - 10.9.1 Design of Phase Shifters, 821
  - 10.9.2 Interfacing of Digital Systems with Different Sampling Rates, 823
  - 10.9.3 Implementation of Narrowband Lowpass Filters, 824
  - 10.9.4 Implementation of Digital Filter Banks, 825
  - 10.9.5 Subband Coding of Speech Signals, 831
  - 10.9.6 Quadrature Mirror Filters, 833
  - 10.9.7 Transmultiplexers, 841
  - 10.9.8 Oversampling A/D and D/A Conversion, 843
- 10.10 Summary and References 844
  - Problems 846

---

<b>11</b>	<b>LINEAR PREDICTION AND OPTIMUM LINEAR FILTERS</b>	<b>852</b>
11.1	Innovations Representation of a Stationary Random Process 852	
11.1.1	Rational Power Spectra, 854	
11.1.2	Relationships Between the Filter Parameters and the Autocorrelation Sequence, 855	
11.2	Forward and Backward Linear Prediction 857	
11.2.1	Forward Linear Prediction, 857	
11.2.2	Backward Linear Prediction, 860	
11.2.3	The Optimum Reflection Coefficients for the Lattice Forward and Backward Predictors, 863	
11.2.4	Relationship of an AR Process to Linear Prediction, 864	
11.3	Solution of the Normal Equations 864	
11.3.1	The Levinson–Durbin Algorithm, 865	
11.3.2	The Schür Algorithm, 868	
11.4	Properties of the Linear Prediction-Error Filters 873	
11.5	AR Lattice and ARMA Lattice-Ladder Filters 876	
11.5.1	AR Lattice Structure, 877	
11.5.2	ARMA Processes and Lattice-Ladder Filters, 878	
11.6	Wiener Filters for Filtering and Prediction 880	
11.6.1	FIR Wiener Filter, 881	
11.6.2	Orthogonality Principle in Linear Mean-Square Estimation, 884	
11.6.3	IIR Wiener Filter, 885	
11.6.4	Noncausal Wiener Filter, 889	
11.7	Summary and References 890	
	Problems 892	
<b>12</b>	<b>POWER SPECTRUM ESTIMATION</b>	<b>896</b>
12.1	Estimation of Spectra from Finite-Duration Observations of Signals 896	
12.1.1	Computation of the Energy Density Spectrum, 897	
12.1.2	Estimation of the Autocorrelation and Power Spectrum of Random Signals: The Periodogram, 902	
12.1.3	The Use of the DFT in Power Spectrum Estimation, 906	
12.2	Nonparametric Methods for Power Spectrum Estimation 908	
12.2.1	The Bartlett Method: Averaging Periodograms, 910	
12.2.2	The Welch Method: Averaging Modified Periodograms, 911	
12.2.3	The Blackman and Tukey Method: Smoothing the Periodogram, 913	
12.2.4	Performance Characteristics of Nonparametric Power Spectrum Estimators, 916	

---

12.2.5	Computational Requirements of Nonparametric Power Spectrum Estimates, 919	
12.3	Parametric Methods for Power Spectrum Estimation 920	
12.3.1	Relationships Between the Autocorrelation and the Model Parameters, 923	
12.3.2	The Yule-Walker Method for the AR Model Parameters, 925	
12.3.3	The Burg Method for the AR Model Parameters, 925	
12.3.4	Unconstrained Least-Squares Method for the AR Model Parameters, 929	
12.3.5	Sequential Estimation Methods for the AR Model Parameters, 930	
12.3.6	Selection of AR Model Order, 931	
12.3.7	MA Model for Power Spectrum Estimation, 933	
12.3.8	ARMA Model for Power Spectrum Estimation, 934	
12.3.9	Some Experimental Results, 936	
12.4	Minimum Variance Spectral Estimation 942	
12.5	Eigenanalysis Algorithms for Spectrum Estimation 946	
12.5.1	Pisarenko Harmonic Decomposition Method, 948	
12.5.2	Eigen-decomposition of the Autocorrelation Matrix for Sinusoids in White Noise, 950	
12.5.3	MUSIC Algorithm, 952	
12.5.4	ESPRIT Algorithm, 953	
12.5.5	Order Selection Criteria, 955	
12.5.6	Experimental Results, 956	
12.6	Summary and References 959	
	Problems 960	
<b>A</b>	<b>RANDOM SIGNALS, CORRELATION FUNCTIONS, AND POWER SPECTRA</b>	<b>A1</b>
<b>B</b>	<b>RANDOM NUMBER GENERATORS</b>	<b>B1</b>
<b>C</b>	<b>TABLES OF TRANSITION COEFFICIENTS FOR THE DESIGN OF LINEAR-PHASE FIR FILTERS</b>	<b>C1</b>
<b>D</b>	<b>LIST OF MATLAB FUNCTIONS</b>	<b>D1</b>
	<b>REFERENCES AND BIBLIOGRAPHY</b>	<b>R1</b>
	<b>INDEX</b>	<b>I1</b>



## Preface

This book was developed based on our teaching of undergraduate and graduate level courses in digital signal processing over the past several years. In this book we present the fundamentals of discrete-time signals, systems, and modern digital processing algorithms and applications for students in electrical engineering, computer engineering, and computer science. The book is suitable for either a one-semester or a two-semester undergraduate level course in discrete systems and digital signal processing. It is also intended for use in a one-semester first-year graduate-level course in digital signal processing.

It is assumed that the student in electrical and computer engineering has had undergraduate courses in advanced calculus (including ordinary differential equations), and linear systems for continuous-time signals, including an introduction to the Laplace transform. Although the Fourier series and Fourier transforms of periodic and aperiodic signals are described in Chapter 4, we expect that many students may have had this material in a prior course.

A balanced coverage is provided of both theory and practical applications. A large number of well designed problems are provided to help the student in mastering the subject matter. A solutions manual is available for the benefit of the instructor and can be obtained from the publisher.

The third edition of the book covers basically the same material as the second edition, but is organized differently. The major difference is in the order in which the DFT and FFT algorithms are covered. Based on suggestions made by several reviewers, we now introduce the DFT and describe its efficient computation immediately following our treatment of Fourier analysis. This reorganization has also allowed us to eliminate repetition of some topics concerning the DFT and its applications.

In Chapter 1 we describe the operations involved in the analog-to-digital conversion of analog signals. The process of sampling a sinusoid is described in some detail and the problem of aliasing is explained. Signal quantization and digital-to-analog conversion are also described in general terms, but the analysis is presented in subsequent chapters.

Chapter 2 is devoted entirely to the characterization and analysis of linear time-invariant (shift-invariant) discrete-time systems and discrete-time signals in the time domain. The convolution sum is derived and systems are categorized according to the duration of their impulse response as a finite-duration impulse

response (FIR) and as an infinite-duration impulse response (IIR). Linear time-invariant systems characterized by difference equations are presented and the solution of difference equations with initial conditions is obtained. The chapter concludes with a treatment of discrete-time correlation.

The  $z$ -transform is introduced in Chapter 3. Both the bilateral and the unilateral  $z$ -transforms are presented, and methods for determining the inverse  $z$ -transform are described. Use of the  $z$ -transform in the analysis of linear time-invariant systems is illustrated, and important properties of systems, such as causality and stability, are related to  $z$ -domain characteristics.

Chapter 4 treats the analysis of signals and systems in the frequency domain. Fourier series and the Fourier transform are presented for both continuous-time and discrete-time signals. Linear time-invariant (LTI) discrete systems are characterized in the frequency domain by their frequency response function and their response to periodic and aperiodic signals is determined. A number of important types of discrete-time systems are described, including resonators, notch filters, comb filters, all-pass filters, and oscillators. The design of a number of simple FIR and IIR filters is also considered. In addition, the student is introduced to the concepts of minimum-phase, mixed-phase, and maximum-phase systems and to the problem of deconvolution.

The DFT, its properties and its applications, are the topics covered in Chapter 5. Two methods are described for using the DFT to perform linear filtering. The use of the DFT to perform frequency analysis of signals is also described.

Chapter 6 covers the efficient computation of the DFT. Included in this chapter are descriptions of radix-2, radix-4, and split-radix fast Fourier transform (FFT) algorithms, and applications of the FFT algorithms to the computation of convolution and correlation. The Goertzel algorithm and the chirp- $z$  transform are introduced as two methods for computing the DFT using linear filtering.

Chapter 7 treats the realization of IIR and FIR systems. This treatment includes direct-form, cascade, parallel, lattice, and lattice-ladder realizations. The chapter includes a treatment of state-space analysis and structures for discrete-time systems, and examines quantization effects in a digital implementation of FIR and IIR systems.

Techniques for design of digital FIR and IIR filters are presented in Chapter 8. The design techniques include both direct design methods in discrete time and methods involving the conversion of analog filters into digital filters by various transformations. Also treated in this chapter is the design of FIR and IIR filters by least-squares methods.

Chapter 9 focuses on the sampling of continuous-time signals and the reconstruction of such signals from their samples. In this chapter, we derive the sampling theorem for bandpass continuous-time-signals and then cover the A/D and D/A conversion techniques, including oversampling A/D and D/A converters.

Chapter 10 provides an indepth treatment of sampling-rate conversion and its applications to multirate digital signal processing. In addition to describing decimation and interpolation by integer factors, we present a method of sampling-rate

conversion by an arbitrary factor. Several applications to multirate signal processing are presented, including the implementation of digital filters, subband coding of speech signals, transmultiplexing, and oversampling A/D and D/A converters.

Linear prediction and optimum linear (Wiener) filters are treated in Chapter 11. Also included in this chapter are descriptions of the Levinson–Durbin algorithm and Schür algorithm for solving the normal equations, as well as the AR lattice and ARMA lattice-ladder filters.

Power spectrum estimation is the main topic of Chapter 12. Our coverage includes a description of nonparametric and model-based (parametric) methods. Also described are eigen-decomposition-based methods, including MUSIC and ESPRIT.

At Northeastern University, we have used the first six chapters of this book for a one-semester (junior level) course in discrete systems and digital signal processing.

A one-semester senior level course for students who have had prior exposure to discrete systems can use the material in Chapters 1 through 4 for a quick review and then proceed to cover Chapter 5 through 8.

In a first-year graduate level course in digital signal processing, the first five chapters provide the student with a good review of discrete-time systems. The instructor can move quickly through most of this material and then cover Chapters 6 through 9, followed by either Chapters 10 and 11 or by Chapters 11 and 12.

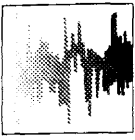
We have included many examples throughout the book and approximately 500 homework problems. Many of the homework problems can be solved numerically on a computer, using a software package such as MATLAB®. These problems are identified by an asterisk. Appendix D contains a list of MATLAB functions that the student can use in solving these problems. The instructor may also wish to consider the use of a supplementary book that contains computer based exercises, such as the books *Digital Signal Processing Using MATLAB* (P.W.S. Kent, 1996) by V. K. Ingle and J. G. Proakis and *Computer-Based Exercises for Signal Processing Using MATLAB* (Prentice Hall, 1994) by C. S. Burrus et al.

The authors are indebted to their many faculty colleagues who have provided valuable suggestions through reviews of the first and second editions of this book. These include Drs. W. E. Alexander, Y. Bresler, J. Deller, V. Ingle, C. Keller, H. Lev-Ari, L. Merakos, W. Mikhael, P. Monticciolo, C. Nikias, M. Schetzen, H. Trussell, S. Wilson, and M. Zoltowski. We are also indebted to Dr. R. Price for recommending the inclusion of split-radix FFT algorithms and related suggestions. Finally, we wish to acknowledge the suggestions and comments of many former graduate students, and especially those by A. L. Kok, J. Lin and S. Srinidhi who assisted in the preparation of several illustrations and the solutions manual.

John G. Proakis  
Dimitris G. Manolakis







# 1

## Introduction

---

Digital signal processing is an area of science and engineering that has developed rapidly over the past 30 years. This rapid development is a result of the significant advances in digital computer technology and integrated-circuit fabrication. The digital computers and associated digital hardware of three decades ago were relatively large and expensive and, as a consequence, their use was limited to general-purpose non-real-time (off-line) scientific computations and business applications. The rapid developments in integrated-circuit technology, starting with medium-scale integration (MSI) and progressing to large-scale integration (LSI), and now, very-large-scale integration (VLSI) of electronic circuits has spurred the development of powerful, smaller, faster, and cheaper digital computers and special-purpose digital hardware. These inexpensive and relatively fast digital circuits have made it possible to construct highly sophisticated digital systems capable of performing complex digital signal processing functions and tasks, which are usually too difficult and/or too expensive to be performed by analog circuitry or analog signal processing systems. Hence many of the signal processing tasks that were conventionally performed by analog means are realized today by less expensive and often more reliable digital hardware.

We do not wish to imply that digital signal processing is the proper solution for all signal processing problems. Indeed, for many signals with extremely wide bandwidths, real-time processing is a requirement. For such signals, analog or, perhaps, optical signal processing is the only possible solution. However, where digital circuits are available and have sufficient speed to perform the signal processing, they are usually preferable.

Not only do digital circuits yield cheaper and more reliable systems for signal processing, they have other advantages as well. In particular, digital processing hardware allows programmable operations. Through software, one can more easily modify the signal processing functions to be performed by the hardware. Thus digital hardware and associated software provide a greater degree of flexibility in system design. Also, there is often a higher order of precision achievable with digital hardware and software compared with analog circuits and analog signal processing systems. For all these reasons, there has been an explosive growth in digital signal processing theory and applications over the past three decades.

In this book our objective is to present an introduction of the basic analysis tools and techniques for digital processing of signals. We begin by introducing some of the necessary terminology and by describing the important operations associated with the process of converting an analog signal to digital form suitable for digital processing. As we shall see, digital processing of analog signals has some drawbacks. First, and foremost, conversion of an analog signal to digital form, accomplished by sampling the signal and quantizing the samples, results in a distortion that prevents us from reconstructing the original analog signal from the quantized samples. Control of the amount of this distortion is achieved by proper choice of the sampling rate and the precision in the quantization process. Second, there are finite precision effects that must be considered in the digital processing of the quantized samples. While these important issues are considered in some detail in this book, the emphasis is on the analysis and design of digital signal processing systems and computational techniques.

## 1.1 SIGNALS, SYSTEMS, AND SIGNAL PROCESSING

A *signal* is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables. For example, the functions

$$\begin{aligned} s_1(t) &= 5t \\ s_2(t) &= 20t^2 \end{aligned} \quad (1.1.1)$$

describe two signals, one that varies linearly with the independent variable  $t$  (time) and a second that varies quadratically with  $t$ . As another example, consider the function

$$s(x, y) = 3x + 2xy + 10y^2 \quad (1.1.2)$$

This function describes a signal of two independent variables  $x$  and  $y$  that could represent the two spatial coordinates in a plane.

The signals described by (1.1.1) and (1.1.2) belong to a class of signals that are precisely defined by specifying the functional dependence on the independent variable. However, there are cases where such a functional relationship is unknown or too highly complicated to be of any practical use.

For example, a speech signal (see Fig. 1.1) cannot be described functionally by expressions such as (1.1.1). In general, a segment of speech may be represented to a high degree of accuracy as a sum of several sinusoids of different amplitudes and frequencies, that is, as

$$\sum_{i=1}^N A_i(t) \sin[2\pi F_i(t)t + \theta_i(t)] \quad (1.1.3)$$

where  $\{A_i(t)\}$ ,  $\{F_i(t)\}$ , and  $\{\theta_i(t)\}$  are the sets of (possibly time-varying) amplitudes, frequencies, and phases, respectively, of the sinusoids. In fact, one way to interpret the information content or message conveyed by any short time segment of the

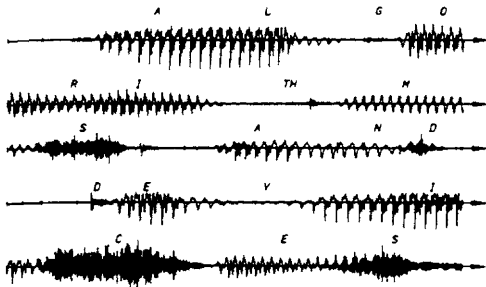


Figure 1.1 Example of a speech signal.

speech signal is to measure the amplitudes, frequencies, and phases contained in the short time segment of the signal.

Another example of a natural signal is an electrocardiogram (ECG). Such a signal provides a doctor with information about the condition of the patient's heart. Similarly, an electroencephalogram (EEG) signal provides information about the activity of the brain.

Speech, electrocardiogram, and electroencephalogram signals are examples of information-bearing signals that evolve as functions of a single independent variable, namely, time. An example of a signal that is a function of two independent variables is an image signal. The independent variables in this case are the spatial coordinates. These are but a few examples of the countless number of natural signals encountered in practice.

Associated with natural signals are the means by which such signals are generated. For example, speech signals are generated by forcing air through the vocal cords. Images are obtained by exposing a photographic film to a scene or an object. Thus signal generation is usually associated with a *system* that responds to a stimulus or force. In a speech signal, the system consists of the vocal cords and the vocal tract, also called the vocal cavity. The stimulus in combination with the system is called a *signal source*. Thus we have speech sources, images sources, and various other types of signal sources.

A *system* may also be defined as a physical device that performs an operation on a signal. For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal is called a system. In this case the filter performs some operation(s) on the signal, which has the effect of reducing (filtering) the noise and interference from the desired information-bearing signal.

When we pass a signal through a system, as in filtering, we say that we have processed the signal. In this case the processing of the signal involves filtering the noise and interference from the desired signal. In general, the system is characterized by the type of operation that it performs on the signal. For example, if the operation is linear, the system is called linear. If the operation on the signal is nonlinear, the system is said to be nonlinear, and so forth. Such operations are usually referred to as *signal processing*.

For our purposes, it is convenient to broaden the definition of a system to include not only physical devices, but also software realizations of operations on a signal. In digital processing of signals on a digital computer, the operations performed on a signal consist of a number of mathematical operations as specified by a software program. In this case, the program represents an implementation of the system in *software*. Thus we have a system that is realized on a digital computer by means of a sequence of mathematical operations; that is, we have a digital signal processing system realized in software. For example, a digital computer can be programmed to perform digital filtering. Alternatively, the digital processing on the signal may be performed by digital *hardware* (logic circuits) configured to perform the desired specified operations. In such a realization, we have a physical device that performs the specified operations. In a broader sense, a digital system can be implemented as a combination of digital hardware and software, each of which performs its own set of specified operations.

This book deals with the processing of signals by digital means, either in software or in hardware. Since many of the signals encountered in practice are analog, we will also consider the problem of converting an analog signal into a digital signal for processing. Thus we will be dealing primarily with digital systems. The operations performed by such a system can usually be specified mathematically. The method or set of rules for implementing the system by a program that performs the corresponding mathematical operations is called an *algorithm*. Usually, there are many ways or algorithms by which a system can be implemented, either in software or in hardware, to perform the desired operations and computations. In practice, we have an interest in devising algorithms that are computationally efficient, fast, and easily implemented. Thus a major topic in our study of digital signal processing is the discussion of efficient algorithms for performing such operations as filtering, correlation, and spectral analysis.

### 1.1.1 Basic Elements of a Digital Signal Processing System

Most of the signals encountered in science and engineering are analog in nature. That is, the signals are functions of a continuous variable, such as time or space, and usually take on values in a continuous range. Such signals may be processed directly by appropriate analog systems (such as filters or frequency analyzers) or frequency multipliers for the purpose of changing their characteristics or extracting some desired information. In such a case we say that the signal has been processed directly in its analog form, as illustrated in Fig. 1.2. Both the input signal and the output signal are in analog form.

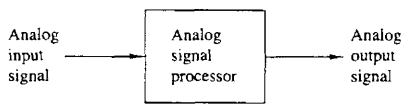


Figure 1.2 Analog signal processing.

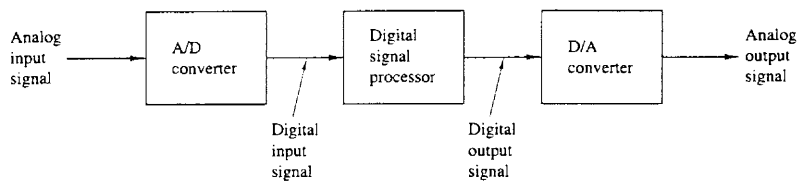


Figure 1.3 Block diagram of a digital signal processing system.

Digital signal processing provides an alternative method for processing the analog signal, as illustrated in Fig. 1.3. To perform the processing digitally, there is a need for an interface between the analog signal and the digital processor. This interface is called an *analog-to-digital (A/D) converter*. The output of the A/D converter is a digital signal that is appropriate as an input to the digital processor.

The digital signal processor may be a large programmable digital computer or a small microprocessor programmed to perform the desired operations on the input signal. It may also be a hardwired digital processor configured to perform a specified set of operations on the input signal. Programmable machines provide the flexibility to change the signal processing operations through a change in the software, whereas hardwired machines are difficult to reconfigure. Consequently, programmable signal processors are in very common use. On the other hand, when signal processing operations are well defined, a hardwired implementation of the operations can be optimized, resulting in a cheaper signal processor and, usually, one that runs faster than its programmable counterpart. In applications where the digital output from the digital signal processor is to be given to the user in analog form, such as in speech communications, we must provide another interface from the digital domain to the analog domain. Such an interface is called a *digital-to-analog (D/A) converter*. Thus the signal is provided to the user in analog form, as illustrated in the block diagram of Fig. 1.3. However, there are other practical applications involving signal analysis, where the desired information is conveyed in digital form and no D/A converter is required. For example, in the digital processing of radar signals, the information extracted from the radar signal, such as the position of the aircraft and its speed, may simply be printed on paper. There is no need for a D/A converter in this case.

### 1.1.2 Advantages of Digital over Analog Signal Processing

There are many reasons why digital signal processing of an analog signal may be preferable to processing the signal directly in the analog domain, as mentioned briefly earlier. First, a digital programmable system allows flexibility in reconfiguring the digital signal processing operations simply by changing the program.

Reconfiguration of an analog system usually implies a redesign of the hardware followed by testing and verification to see that it operates properly.

Accuracy considerations also play an important role in determining the form of the signal processor. Tolerances in analog circuit components make it extremely difficult for the system designer to control the accuracy of an analog signal processing system. On the other hand, a digital system provides much better control of accuracy requirements. Such requirements, in turn, result in specifying the accuracy requirements in the A/D converter and the digital signal processor, in terms of word length, floating-point versus fixed-point arithmetic, and similar factors.

Digital signals are easily stored on magnetic media (tape or disk) without deterioration or loss of signal fidelity beyond that introduced in the A/D conversion. As a consequence, the signals become transportable and can be processed off-line in a remote laboratory. The digital signal processing method also allows for the implementation of more sophisticated signal processing algorithms. It is usually very difficult to perform precise mathematical operations on signals in analog form but these same operations can be routinely implemented on a digital computer using software.

In some cases a digital implementation of the signal processing system is cheaper than its analog counterpart. The lower cost may be due to the fact that the digital hardware is cheaper, or perhaps it is a result of the flexibility for modifications provided by the digital implementation.

As a consequence of these advantages, digital signal processing has been applied in practical systems covering a broad range of disciplines. We cite, for example, the application of digital signal processing techniques in speech processing and signal transmission on telephone channels, in image processing and transmission, in seismology and geophysics, in oil exploration, in the detection of nuclear explosions, in the processing of signals received from outer space, and in a vast variety of other applications. Some of these applications are cited in subsequent chapters.

As already indicated, however, digital implementation has its limitations. One practical limitation is the speed of operation of A/D converters and digital signal processors. We shall see that signals having extremely wide bandwidths require fast-sampling-rate A/D converters and fast digital signal processors. Hence there are analog signals with large bandwidths for which a digital processing approach is beyond the state of the art of digital hardware.

## 1.2 CLASSIFICATION OF SIGNALS

The methods we use in processing a signal or in analyzing the response of a system to a signal depend heavily on the characteristic attributes of the specific signal. There are techniques that apply only to specific families of signals. Consequently, any investigation in signal processing should start with a classification of the signals involved in the specific application.

### 1.2.1 Multichannel and Multidimensional Signals

As explained in Section 1.1, a signal is described by a function of one or more independent variables. The value of the function (i.e., the dependent variable) can be a real-valued scalar quantity, a complex-valued quantity, or perhaps a vector. For example, the signal

$$s_1(t) = A \sin 3\pi t$$

is a real-valued signal. However, the signal

$$s_2(t) = Ae^{j3\pi t} = A \cos 3\pi t + jA \sin 3\pi t$$

is complex valued.

In some applications, signals are generated by multiple sources or multiple sensors. Such signals, in turn, can be represented in vector form. Figure 1.4 shows the three components of a vector signal that represents the ground acceleration due to an earthquake. This acceleration is the result of three basic types of elastic waves. The primary (P) waves and the secondary (S) waves propagate within the body of rock and are longitudinal and transversal, respectively. The third type of elastic wave is called the surface wave, because it propagates near the ground surface. If  $s_k(t)$ ,  $k = 1, 2, 3$ , denotes the electrical signal from the  $k$ th sensor as a function of time, the set of  $p = 3$  signals can be represented by a vector  $\mathbf{S}_3(t)$ , where

$$\mathbf{S}_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

We refer to such a vector of signals as a *multichannel signal*. In electrocardiography, for example, 3-lead and 12-lead electrocardiograms (ECG) are often used in practice, which result in 3-channel and 12-channel signals.

Let us now turn our attention to the independent variable(s). If the signal is a function of a single independent variable, the signal is called a *one-dimensional* signal. On the other hand, a signal is called *M-dimensional* if its value is a function of  $M$  independent variables.

The picture shown in Fig. 1.5 is an example of a two-dimensional signal, since the intensity or brightness  $I(x, y)$  at each point is a function of two independent variables. On the other hand, a black-and-white television picture may be represented as  $I(x, y, t)$  since the brightness is a function of time. Hence the TV picture may be treated as a three-dimensional signal. In contrast, a color TV picture may be described by three intensity functions of the form  $I_r(x, y, t)$ ,  $I_g(x, y, t)$ , and  $I_b(x, y, t)$ , corresponding to the brightness of the three principal colors (red, green, blue) as functions of time. Hence the color TV picture is a three-channel, three-dimensional signal, which can be represented by the vector

$$\mathbf{I}(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t) \end{bmatrix}$$

In this book we deal mainly with single-channel, one-dimensional real- or complex-valued signals and we refer to them simply as signals. In mathematical



- [\*La cabane de l'aiguilleur pdf\*](#)
- [read online Arena Uno: Tratantes de esclavos here](#)
- [download Island Nights' Entertainment](#)
- [read online Best Served Cold](#)
- [The Jazz Language: A Theory Text for Jazz Composition and Improvisation pdf](#)
- [download The New York Times \(08 October 2015\)](#)
  
- <http://growingsomeroots.com/ebooks/Blackbirds.pdf>
- <http://rodrigocaporal.com/library/Betrayed.pdf>
- <http://musor.ruspb.info/?library/The-Amazing-Mrs-Livesey.pdf>
- <http://wind-in-herleshausen.de/?freebooks/Wal-Mart--The-Face-Of-Twenty-First-Century-Capitalism.pdf>
- <http://bestarthritiscare.com/library/Ask-Your-Guides--Connecting-to-Your-Divine-Support-System.pdf>
- <http://korplast.gr/lib/Invisible-Punishment--The-Collateral-Consequences-of-Mass-Imprisonment.pdf>