



Leonard Smith  
**CHAOS**  
A Very Short Introduction

OXFORD

---

## Chaos: A Very Short Introduction

VERY SHORT INTRODUCTIONS are for anyone wanting a stimulating and accessible way in to a new subject. They are written by experts, and have been published in more than 25 languages worldwide.

The series began in 1995, and now represents a wide variety of topics in history, philosophy, religion, science, and the humanities. Over the next few years it will grow to a library of around 200 volumes – a Very Short Introduction to everything from ancient Egypt and Indian philosophy to conceptual art and cosmology.

Very Short Introductions available now:

ANARCHISM Colin Ward	CHOICE THEORY Michael Allingham
ANCIENT EGYPT Ian Shaw	CHRISTIAN ART Beth Williamson
ANCIENT PHILOSOPHY Julia Annas	CHRISTIANITY Linda Woodhead
ANCIENT WARFARE Harry Sidebottom	CLASSICS Mary Beard and John Henderson
ANGLICANISM Mark Chapman	CLAUSEWITZ Michael Howard
THE ANGLO-SAXON AGE John Blair	THE COLD WAR Robert McMahon
ANIMAL RIGHTS David DeGrazia	CONSCIOUSNESS Susan Blackmore
ARCHAEOLOGY Paul Bahn	CONTEMPORARY ART Julian Stallabrass
ARCHITECTURE Andrew Ballantyne	CONTINENTAL PHILOSOPHY Simon Critchley
ARISTOTLE Jonathan Barnes	COSMOLOGY Peter Coles
ART HISTORY Dana Arnold	THE CRUSADES Christopher Tyerman
ART THEORY Cynthia Freeland	CRYPTOGRAPHY Fred Piper and Sean Murphy
THE HISTORY OF ASTRONOMY Michael Hoskin	DADA AND SURREALISM David Hopkins
ATHEISM Julian Baggini	DARWIN Jonathan Howard
AUGUSTINE Henry Chadwick	THE DEAD SEA SCROLLS Timothy Lim
BARTHES Jonathan Culler	DEMOCRACY Bernard Crick
THE BIBLE John Riches	DESCARTES Tom Sorell
THE BRAIN Michael O'Shea	DESIGN John Heskett
BRITISH POLITICS Anthony Wright	DINOSAURS David Norman
BUDDHA Michael Carrithers	DREAMING J. Allan Hobson
BUDDHISM Damien Keown	DRUGS Leslie Iversen
BUDDHIST ETHICS Damien Keown	THE EARTH Martin Redfern
CAPITALISM James Fulcher	ECONOMICS Partha Dasgupta
THE CELTS Barry Cunliffe	EGYPTIAN MYTH Geraldine Pinch
CHAOS Leonard Smith	

---

EIGHTEENTH-CENTURY  
BRITAIN Paul Langford  
THE ELEMENTS Philip Ball  
EMOTION Dylan Evans  
EMPIRE Stephen Howe  
ENGELS Terrell Carver  
ETHICS Simon Blackburn  
THE EUROPEAN UNION  
John Pinder  
EVOLUTION  
Brian and Deborah Charlesworth  
EXISTENTIALISM Thomas Flynn  
FASCISM Kevin Passmore  
FEMINISM Margaret Walters  
THE FIRST WORLD WAR  
Michael Howard  
FOSSILS Keith Thomson  
FOUCAULT Gary Gutting  
THE FRENCH REVOLUTION  
William Doyle  
FREE WILL Thomas Pink  
FREUD Anthony Storr  
FUNDAMENTALISM  
Malise Ruthven  
GALILEO Stillman Drake  
GANDHI Bhikhu Parekh  
GLOBAL CATASTROPHES  
Bill McGuire  
GLOBALIZATION Manfred Steger  
GLOBAL WARMING Mark Maslin  
HABERMAS  
James Gordon Finlayson  
HEGEL Peter Singer  
HEIDEGGER Michael Inwood  
HIEROGLYPHS Penelope Wilson  
HINDUISM Kim Knott  
HISTORY John H. Arnold  
HOBBS Richard Tuck  
HUMAN EVOLUTION  
Bernard Wood  
HUME A. J. Ayer  
IDEOLOGY Michael Freeden  
INDIAN PHILOSOPHY  
Sue Hamilton  
INTELLIGENCE Ian J. Deary  
INTERNATIONAL  
MIGRATION Khalid Koser  
ISLAM Malise Ruthven  
JOURNALISM Ian Hargreaves  
JUDAISM Norman Solomon  
JUNG Anthony Stevens  
KAFKA Ritchie Robertson  
KANT Roger Scruton  
KIERKEGAARD Patrick Gardiner  
THE KORAN Michael Cook  
LINGUISTICS Peter Matthews  
LITERARY THEORY  
Jonathan Culler  
LOCKE John Dunn  
LOGIC Graham Priest  
MACHIAVELLI Quentin Skinner  
THE MARQUIS DE SADE  
John Phillips  
MARX Peter Singer  
MATHEMATICS Timothy Gowers  
MEDICAL ETHICS Tony Hope  
MEDIEVAL BRITAIN  
John Gillingham and  
Ralph A. Griffiths  
MODERN ART David Cottington  
MODERN IRELAND  
Senia Pašeta  
MOLECULES Philip Ball  
MUSIC Nicholas Cook  
MYTH Robert A. Segal  
NATIONALISM Steven Grosby  
NEWTON Robert Iliffe  
NIETZSCHE Michael Tanner  
NINETEENTH-CENTURY  
BRITAIN Christopher Harvie and  
H. C. G. Matthew  
NORTHERN IRELAND  
Marc Mulholland  
PARTICLE PHYSICS Frank Close  
PAUL E. P. Sanders  
PHILOSOPHY Edward Craig  
PHILOSOPHY OF LAW  
Raymond Wacks

---

PHILOSOPHY OF SCIENCE Samir Okasha	SCHIZOPHRENIA Chris Frith and Eve Johnstone
PHOTOGRAPHY Steve Edwards	SCHOPENHAUER Christopher Janaway
PLATO Julia Annas	SHAKESPEARE Germaine Greer
POLITICS Kenneth Minogue	SIKHISM Eleanor Nesbitt
POLITICAL PHILOSOPHY David Miller	SOCIAL AND CULTURAL ANTHROPOLOGY John Monaghan and Peter Just
POSTCOLONIALISM Robert Young	SOCIALISM Michael Newman
POSTMODERNISM Christopher Butler	SOCIOLOGY Steve Bruce
POSTSTRUCTURALISM Catherine Belsey	SOCRATES C. C. W. Taylor
PREHISTORY Chris Gosden	THE SPANISH CIVIL WAR Helen Graham
PRESOCRATIC PHILOSOPHY Catherine Osborne	SPINOZA Roger Scruton
PSYCHOLOGY Gillian Butler and Freda McManus	STUART BRITAIN John Morrill
PSYCHIATRY Tom Burns	TERRORISM Charles Townshend
QUANTUM THEORY John Polkinghorne	THEOLOGY David F. Ford
THE RENAISSANCE Jerry Brotton	THE HISTORY OF TIME Leofranc Holford-Strevens
RENAISSANCE ART Geraldine A. Johnson	TRAGEDY Adrian Poole
ROMAN BRITAIN Peter Salway	THE TUDORS John Guy
THE ROMAN EMPIRE Christopher Kelly	TWENTIETH-CENTURY BRITAIN Kenneth O. Morgan
ROUSSEAU Robert Wokler	THE VIKINGS Julian D. Richards
RUSSELL A. C. Grayling	WITTGENSTEIN A. C. Grayling
RUSSIAN LITERATURE Catriona Kelly	WORLD MUSIC Philip Bohlman
THE RUSSIAN REVOLUTION S. A. Smith	THE WORLD TRADE ORGANIZATION Amrita Narlikar

Available soon:

AFRICAN HISTORY John Parker and Richard Rathbone	HIV/AIDS Alan Whiteside
CHILD DEVELOPMENT Richard Griffin	HUMAN RIGHTS Andrew Clapham
CITIZENSHIP Richard Bellamy	INTERNATIONAL RELATIONS Paul Wilkinson
	RACISM Ali Rattansi

For more information visit our web site  
[www.oup.co.uk/general/vsi/](http://www.oup.co.uk/general/vsi/)

---

Leonard A. Smith

# CHAOS

A Very Short Introduction

**OXFORD**  
UNIVERSITY PRESS

---

# OXFORD

UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.  
It furthers the University's objective of excellence in research, scholarship,  
and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi

Kuala Lumpur Madrid Melbourne Mexico City Nairobi

New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece

Guatemala Hungary Italy Japan Poland Portugal Singapore

South Korea Switzerland Thailand Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press  
in the UK and in certain other countries

Published in the United States  
by Oxford University Press Inc., New York

© Leonard A. Smith 2007

The moral rights of the author have been asserted  
Database right Oxford University Press (maker)

First published as a Very Short Introduction 2007

All rights reserved. No part of this publication may be reproduced,  
stored in a retrieval system, or transmitted, in any form or by any means,  
without the prior permission in writing of Oxford University Press,  
or as expressly permitted by law, or under terms agreed with the appropriate  
reprographics rights organizations. Enquiries concerning reproduction  
outside the scope of the above should be sent to the Rights Department,  
Oxford University Press, at the address above

You must not circulate this book in any other binding or cover  
and you must impose this same condition on any acquirer

British Library Cataloguing in Publication Data  
Data available

Library of Congress Cataloging in Publication Data  
Data available

Typeset by RefineCatch Ltd, Bungay, Suffolk  
Printed in Great Britain by  
Ashford Colour Press Ltd, Gosport, Hampshire

978-0-19-285378-3

1 3 5 7 9 10 8 6 4 2

---

*To the memory of Dave Paul Debeer,  
A real physicist, a true friend.*



---

*This page intentionally left blank*

---

# Contents

	Acknowledgements	xi
	Preface	xii
	List of illustrations	xv
1	The emergence of chaos	1
2	Exponential growth, nonlinearity, common sense	22
3	Chaos in context: determinism, randomness, and noise	33
4	Chaos in mathematical models	58
5	Fractals, strange attractors, and dimension(s)	76
6	Quantifying the dynamics of uncertainty	87
7	Real numbers, real observations, and computers	104
8	Sorry, wrong number: statistics and chaos	112
9	Predictability: does chaos constrain our forecasts?	123
10	Applied chaos: can we see through our models?	132
11	Philosophy in chaos	154
	Glossary	163

---

Further reading 169

Index 173

---

# Acknowledgements

This book would not have been possible without my parents, of course, but I owe a greater debt than most to their faith, doubt, and hope, and to the love and patience of a, b, and c. Professionally my greatest debt is to Ed Spiegel, a father of chaos and my thesis Professor, mentor, and friend. I also profited immensely from having the chance to discuss some of these ideas with Jim Berger, Robert Bishop, David Broomhead, Neil Gordon, Julian Hunt, Kevin Judd, Joe Keller, Ed Lorenz, Bob May, Michael Mackey, Tim Palmer, Itamar Procaccia, Colin Sparrow, James Theiler, John Wheeler, and Christine Ziehmann. I am happy to acknowledge discussions with, and the support of, the Master and Fellows of Pembroke College, Oxford. Lastly and largely, I'd like to acknowledge my debt to my students, they know who they are. I am never sure how to react upon overhearing an exchange like: 'Did you know she was Lenny's student?', 'Oh, that explains a lot.' Sorry guys: blame Spiegel.

---

# Preface

The 'chaos' introduced in the following pages reflects phenomena in mathematics and the sciences, systems where (without cheating) small differences in the way things are now have huge consequences in the way things will be in the future. It would be cheating, of course, if things just happened randomly, or if everything continually exploded forever. This book traces out the remarkable richness that follows from three simple constraints, which we'll call *sensitivity*, *determinism*, and *recurrence*. These constraints allow mathematical chaos: behaviour that looks random, but is not random. When allowed a bit of *uncertainty*, presumed to be the active ingredient of forecasting, chaos has reignited a centuries-old debate on the nature of the world.

The book is self-contained, defining these terms as they are encountered. My aim is to show the what, where, and how of chaos; sidestepping any topics of 'why' which require an advanced mathematical background. Luckily, the description of chaos and forecasting lends itself to a visual, geometric understanding; our examination of chaos will take us to the coalface of predictability without equations, revealing open questions of active scientific research into the weather, climate, and other real-world phenomena of interest.

Recent popular interest in the science of chaos has evolved

---

differently than did the explosion of interest in science a century ago when special relativity hit a popular nerve that was to throb for decades. Why was the public reaction to science's embrace of mathematical chaos different? Perhaps one distinction is that most of us already knew that, sometimes, very small differences can have huge effects. The concept now called 'chaos' has its origins both in science fiction and in science fact. Indeed, these ideas were well grounded in fiction before they were accepted as fact: perhaps the public were already well versed in the implications of chaos, while the scientists remained in denial? Great scientists and mathematicians had sufficient courage and insight to foresee the coming of chaos, but until recently mainstream science required a good solution to be well behaved: fractal objects and chaotic curves were considered not only deviant, but the sign of badly posed questions. For a mathematician, few charges carry more shame than the suggestion that one's professional life has been spent on a badly posed question. Some scientists still dislike problems whose results are expected to be irreproducible even in theory. The solutions that chaos requires have only become widely acceptable in scientific circles recently, and the public enjoyed the 'I told you so' glee usually claimed by the 'experts'. This also suggests why chaos, while widely nurtured in mathematics and the sciences, took root within applied sciences like meteorology and astronomy. The applied sciences are driven by a desire to understand and predict reality, a desire that overcame the niceties of whatever the formal mathematics of the day. This required rare individuals who could span the divide between our models of the world and the world as it is without convoluting the two; who could distinguish the mathematics from the reality and thereby extend the mathematics.

As in all *Very Short Introductions*, restrictions on space require entire research programmes to be glossed over or omitted; I present a few recurring themes in context, rather than a series of shallow descriptions. My apologies to those whose work I have omitted, and my thanks to Luciana O'Flaherty (my editor), Wendy Parker, and Lyn Grove for help in distinguishing between what

---

was most interesting to me and what I might make interesting to the reader.

## How to read this introduction

While there is some mathematics in this book, there are no equations more complicated than  $X = 2$ . Jargon is less easy to discard. Words in ***bold italics*** you will have to come to grips with; these are terms that are central to chaos, brief definitions of these words can be found in the Glossary at the end of the book. *Italics* is used both for emphasis and to signal jargon needed for the next page or so, but which is unlikely to recur often throughout the book.

Any questions that haunt you would be welcome online at <http://cats.lse.ac.uk/forum/> on the discussion forum VSI Chaos. More information on these terms can be found rapidly at Wikipedia <http://www.wikipedia.org/> and <http://cats.lse.ac.uk/predictability-wiki/>, and in the Further reading.

---

# List of illustrations

- |   |   |    |    |  |    |
|---|---|----|----|--|----|
| 1 | The first weather map ever published in a newspaper, prepared by Galton in 1875<br>© The Times/NI Syndication Limited             | 7  | 6  | A graph comparing Fibonacci numbers and exponential growth   | 26 |
| 2 | Galton's original sketch of the Galton Board  | 9  | 7  | A chaotic time series from the Full Logistic Map             | 39 |
| 3 | <i>The Times</i> headline following the Burns' Day storm in 1990<br>© The Times/NI Syndication Limited 1990/John Frost Newspapers | 13 | 8  | Six mathematical maps  | 40 |
| 4 | Modern weather map showing the Burns' Day storm and a two-day-ahead forecast  | 14 | 9  | Points collapsing onto four attractors of the Logistic Map   | 48 |
| 5 | <i>The Cheat with the Ace of Diamonds</i> , c.1645, by Georges de la Tour<br>Louvre, Paris. © Photo12.com/Oronoz                  | 19 | 10 | The evolution of uncertainty under the Yule Map              | 52 |
|   |   |    | 11 | Period doubling behaviour in the Logistic Map                | 61 |
|   |   |    | 12 | A variety of more complicated behaviours in the Logistic Map | 62 |



---

13	Three-dimensional bifurcation diagram and the collapse toward attractors in the Logistic Map	63	21	Schematic diagrams showing the action of the Baker's Map and a Baker's Apprentice Map	98
14	The Lorenz attractor and the Moore-Spiegel attractor	67	22	Predictable chaos as seen in four iterations of the same mouse ensemble under the Baker's Map and a Baker's Apprentice Map	100
15	The evolution of uncertainty in the Lorenz System	68	23	Card trick revealing the limitations of digital computers	108
16	The Hénon attractor and a two-dimensional slice of the Moore-Spiegel attractor	70	24	Two views of data from Machete's electric circuit, suggestive of Takens' Theorem	118
17	A variety of behaviours from the Hénon-Heilies System	72	25	The Not A Galton Board	128
18	The Fournier Universe, as illustrated by Fournier	78	26	An illustration of using analogues to make a forecast	134
19	Time series from the stochastic Middle Thirds IFS Map and the deterministic Tripling Tent Map	82	27	The state space of a climate model Crown Copyright	136
20	A close look at the Hénon attractor, showing fractal structure	84	28	Richardson's dream © F. Schuiten	137

---

29	Two-day-ahead ECMWF ensemble forecasts of the Burns' Day storm	140	30	Four ensemble forecasts of the Machete's Moore-Spiegel Circuit	150
----	--	-----	----	--	-----

Figures 7, 8, 9, 11, 12, 13, 19, and 20 were produced with the assistance of Hailiang Du. Figures 24 and 30 were produced with the assistance of Reason Machete. Figures 4 and 29 were produced with the assistance of Martin Leutbecher with data kindly made available by the European Centre for Medium-Range Weather Forecasting. Figure 27 is after M. Hume et al., The UKIPO2 Scientific Report, Tyndal Centre, University of East Anglia, Norwich, UK.

The publisher and the author apologize for any errors or omissions in the above list. If contacted they will be pleased to rectify these at the earliest opportunity.

---

*This page intentionally left blank*

---

# Chapter 1

## The emergence of chaos

Embedded in the mud, glistening green and gold and black,  
was a butterfly, very beautiful and very dead.

It fell to the floor, an exquisite thing, a small thing  
that could upset balances and knock down a line of  
small dominoes and then big dominoes and then  
gigantic dominoes, all down the years across Time.

Ray Bradbury (1952)

### Three hallmarks of mathematical chaos

The 'butterfly effect' has become a popular slogan of chaos. But is it really so surprising that minor details sometimes have major impacts? Sometimes the proverbial minor detail is taken to be the difference between a world with some butterfly and an alternative universe that is exactly like the first, except that the butterfly is absent; as a result of this small difference, the worlds soon come to differ dramatically from one another. The mathematical version of this concept is known as *sensitive dependence*. Chaotic systems not only exhibit sensitive dependence, but two other properties as well: they are *deterministic*, and they are *nonlinear*. In this chapter, we'll see what these words mean and how these concepts came into science.

Chaos is important, in part, because it helps us to cope with

---

unstable systems by improving our ability to describe, to understand, perhaps even to forecast them. Indeed, one of the myths of chaos we will debunk is that chaos makes forecasting a useless task. In an alternative but equally popular butterfly story, there is one world where a butterfly flaps its wings and another world where it does not. This small difference means a tornado appears in only one of these two worlds, linking chaos to uncertainty and prediction: in which world are we? Chaos is the name given to the mechanism which allows such rapid growth of uncertainty in our mathematical models. The image of chaos amplifying uncertainty and confounding forecasts will be a recurring theme throughout this Introduction.

## Whispers of chaos

Warnings of chaos are everywhere, even in the nursery. The warning that a kingdom could be lost for the want of a nail can be traced back to the 14th century; the following version of the familiar nursery rhyme was published in *Poor Richard's Almanack* in 1758 by Benjamin Franklin:

For want of a nail the shoe was lost,  
For want of a shoe the horse was lost,  
and for want of a horse the rider was lost,  
being overtaken and slain by the enemy,  
all for the want of a horse-shoe nail.

We do not seek to explain the seed of instability with chaos, but rather to describe the growth of uncertainty *after* the initial seed is sown. In this case, explaining how it came to be that the rider was lost due to a missing nail, not the fact that the nail had gone missing. In fact, of course, there either was a nail or there was not. But Poor Richard tells us that if the nail hadn't been lost, then the kingdom wouldn't have been lost either. We will often explore the properties of chaotic systems by considering the impact of slightly different situations.

---

The study of chaos is common in applied sciences like astronomy, meteorology, population biology, and economics. Sciences making accurate observations of the world along with quantitative predictions have provided the main players in the development of chaos since the time of Isaac Newton. According to Newton's Laws, the future of the solar system is completely determined by its current state. The 19th-century scientist Pierre Laplace elevated this determinism to a key place in science. A world is deterministic if its current state completely defines its future. In 1820, Laplace conjured up an entity now known as 'Laplace's demon'; in doing so, he linked determinism and the ability to predict in principle to the very notion of success in science.

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

Note that Laplace had the foresight to give his demon three properties: exact knowledge of the Laws of Nature ('all the forces'), the ability to take a snapshot of the exact state of the universe ('all the positions'), and infinite computational resources ('an intellect vast enough to submit these data to analysis'). For Laplace's demon, chaos poses no barrier to prediction. Throughout this Introduction, we will consider the impact of removing one or more of these gifts.

From the time of Newton until the close of the 19th century, most scientists were also meteorologists. Chaos and meteorology are closely linked by the meteorologists' interest in the role uncertainty plays in weather forecasts. Benjamin Franklin's interest in

---

meteorology extended far beyond his famous experiment of flying a kite in a thunderstorm. He is credited with noting the general movement of the weather from west towards the east and testing this theory by writing letters from Philadelphia to cities further east. Although the letters took longer to arrive than the weather, these are arguably early weather forecasts. Laplace himself discovered the law describing the decrease of atmospheric pressure with height. He also made fundamental contributions to the theory of errors: when we make an observation, the measurement is never exact in a mathematical sense, so there is always some uncertainty as to the 'True' value. Scientists often say that any uncertainty in an observation is due to *noise*, without really defining exactly what the noise is, other than that which obscures our vision of whatever we are trying to measure, be it the length of a table, the number of rabbits in a garden, or the midday temperature. Noise gives rise to *observational uncertainty*, chaos helps us to understand how small uncertainties can become large uncertainties, once we have a model for the noise. Some of the insights gleaned from chaos lie in clarifying the role(s) noise plays in the dynamics of uncertainty in the quantitative sciences. Noise has become much more interesting, as the study of chaos forces us to look again at what we might mean by the concept of a 'True' value.

Twenty years after Laplace's book on probability theory appeared, Edgar Allan Poe provided an early reference to what we would now call chaos in the atmosphere. He noted that merely moving our hands would affect the atmosphere all the way around the planet. Poe then went on to echo Laplace, stating that the mathematicians of the Earth could compute the progress of this hand-waving 'impulse', as it spread out and forever altered the state of the atmosphere. Of course, it is up to us whether or not we choose to wave our hands: free will offers another source of seeds that chaos might nurture.

In 1831, between the publication of Laplace's science and Poe's

---

fiction, Captain Robert Fitzroy took the young Charles Darwin on his voyage of discovery. The observations made on this voyage led Darwin to his theory of natural selection. Evolution and chaos have more in common than one might think. First, when it comes to language, both 'evolution' and 'chaos' are used simultaneously to refer both to phenomena to be explained and to the theories that are supposed to do the explaining. This often leads to confusion between the description and the object described (as in 'confusing the map with the territory'). Throughout this Introduction we will see that confusing our mathematical models with the reality they aim to describe muddles the discussion of both. Second, looking more deeply, it may be that some ecosystems evolve as if they were chaotic systems, as it may well be the case that small differences in the environment have immense impacts. And evolution has contributed to the discussion of chaos as well. This chapter's opening quote comes from Ray Bradbury's 'A Sound Like Thunder', in which time-travelling big game hunters accidentally kill a butterfly, and find the future a different place when they return to it. The characters in the story imagine the impact of killing a mouse, its death cascading through generations of lost mice, foxes, and lions, and:

The emergence of chaos

all manner of insects, vultures, infinite billions of life forms are thrown into chaos and destruction . . . Step on a mouse and you leave your print, like a Grand Canyon, across Eternity. Queen Elizabeth might never be born, Washington might not cross the Delaware, there might never be a United States at all. So be careful. Stay on the Path. Never step off!

Needless to say, someone does step off the Path, crushing to death a beautiful little green and black butterfly. We can only consider these 'what if' experiments within the fictions of mathematics or literature, since we have access to only one realization of reality.

The origins of the term 'butterfly effect' are appropriately shrouded



---

sample content of Chaos: A Very Short Introduction

- [read online Hallowe'en Party \(Hercule Poirot, Book 36\)](#)
- [Prime Witness \(A Paul Madriani Novel\) here](#)
- [Doubled Flowering: From the Notebooks of Araki Yasusada pdf, azw \(kindle\), epub](#)
- [click Isengard and Northern Gondor \(MERP ICE #2800\)](#)
  
- <http://www.experienceolvera.co.uk/library/1688--The-First-Modern-Revolution--The-Lewis-Walpole-Series-in-Eighteenth-Century-Culture-and-History-.pdf>
- <http://cavalldecartro.highlandagency.es/library/Performing-Marx--Contemporary-Negotiations-of-a-Living-Tradition--SUNY-Series-in-Political-Theory-.pdf>
- <http://www.celebritychat.in/?ebooks/Doubled-Flowering--From-the-Notebooks-of-Araki-Yasusada.pdf>
- <http://cambridgebrass.com/?freebooks/Isengard-and-Northern-Gondor--MERP-ICE--2800-.pdf>