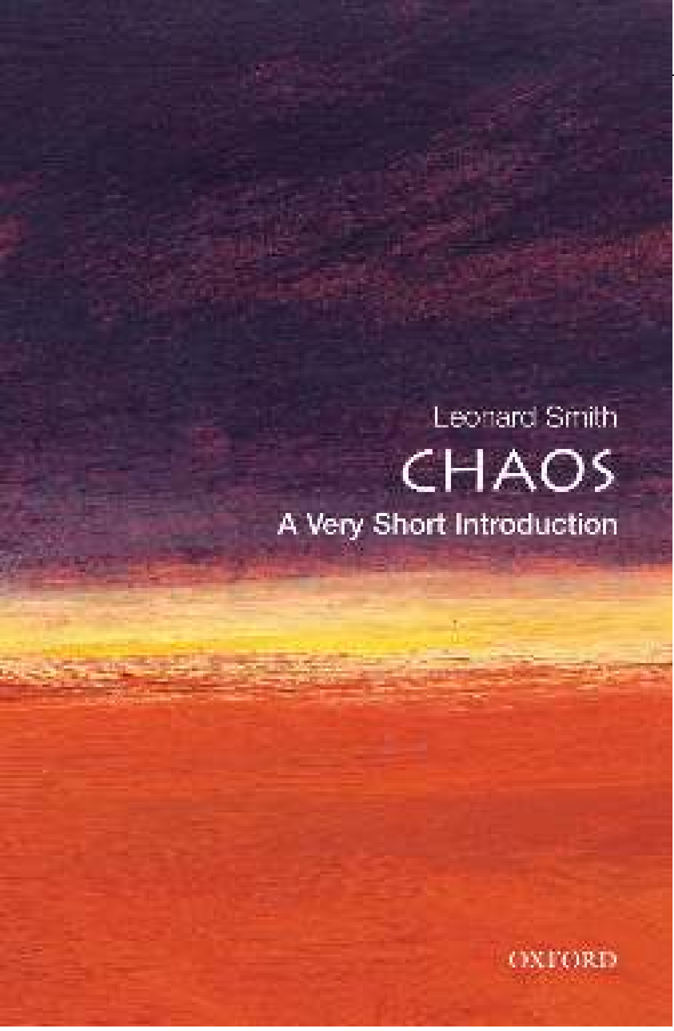
Leonard Smith
CHAOS
A Very Short Introduction
OXFORD

Chaos: A Very Short Introduction

VERY SHORT INTRODUCTIONS are for anyone wanting a stimulating and accessible way in to a new subject. They are written by experts, and have been published in more than 25 languages worldwide.

The series began in 1995, and now represents a wide variety of topics in history, philosophy, religion, science, and the humanities. Over the next few years it will grow to a library of around 200 volumes – a Very Short Introduction to everything from ancient Egypt and Indian philosophy to conceptual art and cosmology.

Very Short Introductions available now:

ANARCHISM Colin Ward
ANCIENT EGYPT Ian Shaw
ANCIENT PHILOSOPHY Julia Annas
ANCIENT WARFARE Harry Sidebottom
ANGLICANISM Mark Chapman
THE ANGLO-SAXON AGE John Blair
ANIMAL RIGHTS David DeGrazia
ARCHAEOLOGY Paul Bahn
ARCHITECTURE Andrew Ballantyne
ARISTOTLE Jonathan Barnes
ART HISTORY Dana Arnold
ART THEORY Cynthia Freeland
THE HISTORY OF ASTRONOMY Michael Hoskin
ATHEISM Julian Baggini
AUGUSTINE Henry Chadwick
BARTHES Jonathan Culler
THE BIBLE John Riches
THE BRAIN Michael O'Shea
BRITISH POLITICS Anthony Wright
BUDDHA Michael Carrithers
BUDDHISM Damien Keown
BUDDHIST ETHICS Damien Keown
CAPITALISM James Fulcher
THE CELTS Barry Cunliffe
CHAOS Leonard Smith
CHOICE THEORY Michael Allingham
CHRISTIAN ART Beth Williamson
CHRISTIANITY Linda Woodhead
CLASSICS Mary Beard and John Henderson
CLAUSEWITZ Michael Howard
THE COLD WAR Robert McMahon
CONSCIOUSNESS Susan Blackmore
CONTEMPORARY ART Julian Stallabrass
CONTINENTAL PHILOSOPHY Simon Critchley
COSMOLOGY Peter Coles
THE CRUSADES Christopher Tyerman
CRYPTOGRAPHY Fred Piper and Sean Murphy
DADA AND SURREALISM David Hopkins
DARWIN Jonathan Howard
THE DEAD SEA SCROLLS Timothy Lim
DEMOCRACY Bernard Crick
DESCARTES Tom Sorell
DESIGN John Heskett
DINOSAURS David Norman
DREAMING J. Allan Hobson
DRUGS Leslie Iversen
THE EARTH Martin Redfern
ECONOMICS Partha Dasgupta
EGYPTIAN MYTH Geraldine Pinch

EIGHTEENTH-CENTURY BRITAIN Paul Langford
THE ELEMENTS Philip Ball
EMOTION Dylan Evans
EMPIRE Stephen Howe
ENGELS Terrell Carver
ETHICS Simon Blackburn
THE EUROPEAN UNION John Pinder
EVOLUTION Brian and Deborah Charlesworth
EXISTENTIALISM Thomas Flynn
FASCISM Kevin Passmore
FEMINISM Margaret Walters
THE FIRST WORLD WAR Michael Howard
FOSSILS Keith Thomson
FOUCAULT Gary Gutting
THE FRENCH REVOLUTION William Doyle
FREE WILL Thomas Pink
FREUD Anthony Storr
FUNDAMENTALISM Malise Ruthven
GALILEO Stillman Drake
GANDHI Bhikhu Parekh
GLOBAL CATASTROPHES Bill McGuire
GLOBALIZATION Manfred Steger
GLOBAL WARMING Mark Maslin
HABERMAS James Gordon Finlayson
HEGEL Peter Singer
HEIDEGGER Michael Inwood
HIEROGLYPHS Penelope Wilson
HINDUISM Kim Knott
HISTORY John H. Arnold
HOBBES Richard Tuck
HUMAN EVOLUTION Bernard Wood
HUME A. J. Ayer
IDEOLOGY Michael Freeden
INDIAN PHILOSOPHY Sue Hamilton
INTELLIGENCE Ian J. Deary
INTERNATIONAL MIGRATION Khalid Koser
ISLAM Malise Ruthven
JOURNALISM Ian Hargreaves
JUDAISM Norman Solomon
JUNG Anthony Stevens
KAFKA Ritchie Robertson
KANT Roger Scruton
KIERKEGAARD Patrick Gardiner
THE KORAN Michael Cook
LINGUISTICS Peter Matthews
LITERARY THEORY Jonathan Culler
LOCKE John Dunn
LOGIC Graham Priest
MACHIAVELLI Quentin Skinner
THE MARQUIS DE SADE John Phillips
MARX Peter Singer
MATHEMATICS Timothy Gowers
MEDICAL ETHICS Tony Hope
MEDIEVAL BRITAIN John Gillingham and Ralph A. Griffiths
MODERN ART David Cottington
MODERN IRELAND Senia Pašeta
MOLECULES Philip Ball
MUSIC Nicholas Cook
MYTH Robert A. Segal
NATIONALISM Steven Grosby
NEWTON Robert Iliffe
NIETZSCHE Michael Tanner
NINETEENTH-CENTURY BRITAIN Christopher Harvie and H. C. G. Matthew
NORTHERN IRELAND Marc Mulholland
PARTICLE PHYSICS Frank Close
PAUL E. P. Sanders
PHILOSOPHY Edward Craig
PHILOSOPHY OF LAW Raymond Wacks

PHILOSOPHY OF SCIENCE Samir Okasha
PHOTOGRAPHY Steve Edwards
PLATO Julia Annas
POLITICS Kenneth Minogue
POLITICAL PHILOSOPHY David Miller
POSTCOLONIALISM Robert Young
POSTMODERNISM Christopher Butler
POSTSTRUCTURALISM Catherine Belsey
PREHISTORY Chris Gosden
PRESOCRATIC PHILOSOPHY Catherine Osborne
PSYCHOLOGY Gillian Butler and Freda McManus
PSYCHIATRY Tom Burns
QUANTUM THEORY John Polkinghorne
THE RENAISSANCE Jerry Brotton
RENAISSANCE ART Geraldine A. Johnson
ROMAN BRITAIN Peter Salway
THE ROMAN EMPIRE Christopher Kelly
ROUSSEAU Robert Wokler
RUSSELL A. C. Grayling
RUSSIAN LITERATURE Catriona Kelly
THE RUSSIAN REVOLUTION S. A. Smith
SCHIZOPHRENIA Chris Frith and Eve Johnstone
SCHOPENHAUER Christopher Janaway
SHAKESPEARE Germaine Greer
SIKHISM Eleanor Nesbitt
SOCIAL AND CULTURAL ANTHROPOLOGY John Monaghan and Peter Just
SOCIALISM Michael Newman
SOCIOLOGY Steve Bruce
SOCRATES C. C. W. Taylor
THE SPANISH CIVIL WAR Helen Graham
SPINOZA Roger Scruton
STUART BRITAIN John Morrill
TERRORISM Charles Townshend
THEOLOGY David F. Ford
THE HISTORY OF TIME Leofranc Holford-Strevens
TRAGEDY Adrian Poole
THE TUDORS John Guy
TWENTIETH-CENTURY BRITAIN Kenneth O. Morgan
THE VIKINGS Julian D. Richards
WITTGENSTEIN A. C. Grayling
WORLD MUSIC Philip Bohlman
THE WORLD TRADE ORGANIZATION Amrita Narlikar

Available soon:

AFRICAN HISTORY John Parker and Richard Rathbone
CHILD DEVELOPMENT Richard Griffin
CITIZENSHIP Richard Bellamy
HIV/AIDS Alan Whiteside
HUMAN RIGHTS Andrew Clapham
INTERNATIONAL RELATIONS Paul Wilkinson
RACISM Ali Rattansi

For more information visit our web site
www.oup.co.uk/general/vsi/

Leonard A. Smith

CHAOS

A Very Short Introduction

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi
Kuala Lumpur Madrid Melbourne Mexico City Nairobi
New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece
Guatemala Hungary Italy Japan Poland Portugal Singapore
South Korea Switzerland Thailand Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press
in the UK and in certain other countries

Published in the United States
by Oxford University Press Inc., New York

First published as a Very Short Introduction 2007

British Library Cataloguing in Publication Data
Data available

Library of Congress Cataloging in Publication Data
Data available

Typeset by RefineCatch Ltd, Bungay, Suffolk
Printed in Great Britain by
Ashford Colour Press Ltd, Gosport, Hampshire

978–0–19–285378–3

1 3 5 7 9 10 8 6 4 2

To the memory of Dave Paul Debeer,
A real physicist, a true friend.

This page intentionally left blank

Contents

Acknowledgements

This book would not have been possible without my parents, of course, but I owe a greater debt than most to their faith, doubt, and hope, and to the love and patience of a, b, and c. Professionally my greatest debt is to Ed Spiegel, a father of chaos and my thesis Professor, mentor, and friend. I also profited immensely from having the chance to discuss some of these ideas with Jim Berger, Robert Bishop, David Broomhead, Neil Gordon, Julian Hunt, Kevin Judd, Joe Keller, Ed Lorenz, Bob May, Michael Mackey, Tim Palmer, Itamar Procaccia, Colin Sparrow, James Theiler, John Wheeler, and Christine Ziehmann. I am happy to acknowledge discussions with, and the support of, the Master and Fellows of Pembroke College, Oxford. Lastly and largely, I'd like to acknowledge my debt to my students, they know who they are. I am never sure how to react upon overhearing an exchange like: 'Did you know she was Lenny's student?', 'Oh, that explains a lot.' Sorry guys: blame Spiegel.

Preface

The 'chaos' introduced in the following pages reflects phenomena in mathematics and the sciences, systems where (without cheating) small differences in the way things are now have huge consequences in the way things will be in the future. It would be cheating, of course, if things just happened randomly, or if everything continually exploded forever. This book traces out the remarkable richness that follows from three simple constraints, which we'll call *sensitivity*, *determinism*, and *recurrence*. These constraints allow mathematical chaos: behaviour that looks random, but is not random. When allowed a bit of *uncertainty*, presumed to be the active ingredient of forecasting, chaos has reignited a centuries-old debate on the nature of the world.

The book is self-contained, defining these terms as they are encountered. My aim is to show the what, where, and how of chaos; sidestepping any topics of 'why' which require an advanced mathematical background. Luckily, the description of chaos and forecasting lends itself to a visual, geometric understanding; our examination of chaos will take us to the coalface of predictability without equations, revealing open questions of active scientific research into the weather, climate, and other real-world phenomena of interest.

Recent popular interest in the science of chaos has evolved differently than did the explosion of interest in science a century ago when special relativity hit a popular nerve that was to throb for decades. Why was the public reaction to science's embrace of mathematical chaos different? Perhaps one distinction is that most of us already knew that, sometimes, very small differences can have huge effects. The concept now called 'chaos' has its origins both in science fiction and in science fact. Indeed, these ideas were well grounded in fiction before they were accepted as fact: perhaps the public were already well versed in the implications of chaos, while the scientists remained in denial? Great scientists and mathematicians had sufficient courage and insight to foresee the coming of chaos, but until recently mainstream science required a good solution to be well behaved: fractal objects and chaotic curves were considered not only deviant, but the sign of badly posed questions. For a mathematician, few charges carry more shame than the suggestion that one's professional life has been spent on a badly posed question. Some scientists still dislike problems whose results are expected to be irreproducible even in theory. The solutions that chaos requires have only become widely acceptable in scientific circles recently, and the public enjoyed the 'I told you so' glee usually claimed by the 'experts'. This also suggests why chaos, while widely nurtured in mathematics and the sciences, took root within applied sciences like meteorology and astronomy. The applied sciences are driven by a desire to understand and predict reality, a desire that overcame the niceties of whatever the formal mathematics of the day. This required rare individuals who could span the divide between our models of the world and the world as it is without convoluting the two; who could distinguish the mathematics from the reality and thereby extend the mathematics.

As in all *Very Short Introductions*, restrictions on space require entire research programmes to be glossed over or omitted; I present a few recurring themes in context, rather than a series of shallow descriptions. My apologies to those whose work I have omitted, and my thanks to Luciana O'Flaherty (my editor), Wendy Parker, and Lyn Grove for help in distinguishing between what was most interesting to me and what I might make interesting to the reader.

How to read this introduction

While there is some mathematics in this book, there are no equations more complicated than X = 2. Jargon is less easy to discard. Words in ***bold italics*** you will have to come to grips with; these are terms that are central to chaos, brief definitions of these words can be found in the Glossary at the end of the book. *Italics* is used both for emphasis and to signal jargon needed for the next page or so, but which is unlikely to recur often throughout the book.

Any questions that haunt you would be welcome online at *http://cats.lse.ac.uk/forum/* on the discussion forum VSI Chaos. More information on these terms can be found rapidly at Wikipedia *http://www.wikipedia.org/* and *http://cats.lse.ac.uk/preditcability-wiki/* , and in the Further reading.

List of illustrations

Figures 7, 8, 9, 11, 12, 13, 19, and 20 were produced with the assistance of Hailiang Du. Figures 24 and 30 were produced with the assistance of Reason Machete. Figures 4 and 29 were produced with the assistance of Martin Leutbecher with data kindly made available by the European Centre for Medium-Range Weather Forecasting. Figure 27 is after M. Hume et al., The UKIP02 Scientific Report, Tyndal Centre, University of East Anglia, Norwich, UK.

The publisher and the author apologize for any errors or omissions in the above list. If contacted they will be pleased to rectify these at the earliest opportunity.

This page intentionally left blank

Chapter 1
The emergence of chaos

> Embedded in the mud, glistening green and gold and black,
> was a butterfly, very beautiful and very dead.
> It fell to the floor, an exquisite thing, a small thing
> that could upset balances and knock down a line of
> small dominoes and then big dominoes and then
> gigantic dominoes, all down the years across Time.
>
> Ray Bradbury (1952)

Three hallmarks of mathematical chaos

The 'butterfly effect' has become a popular slogan of chaos. But is it really so surprising that minor details sometimes have major impacts? Sometimes the proverbial minor detail is taken to be the difference between a world with some butterfly and an alternative universe that is exactly like the first, except that the butterfly is absent; as a result of this small difference, the worlds soon come to differ dramatically from one another. The mathematical version of this concept is known as ***sensitive dependence***. Chaotic systems not only exhibit sensitive dependence, but two other properties as well: they are ***deterministic***, and they are ***nonlinear***. In this chapter, we'll see what these words mean and how these concepts came into science.

Chaos is important, in part, because it helps us to cope with unstable systems by improving our ability to describe, to understand, perhaps even to forecast them. Indeed, one of the myths of chaos we will debunk is that chaos makes forecasting a useless task. In an alternative but equally popular butterfly story, there is one world where a butterfly flaps its wings and another world where it does not. This small difference means a tornado appears in only one of these two worlds, linking chaos to uncertainty and prediction: in which world are we? Chaos is the name given to the mechanism which allows such rapid growth of uncertainty in our mathematical models. The image of chaos amplifying uncertainty and confounding forecasts will be a recurring theme throughout this Introduction.

Whispers of chaos

Warnings of chaos are everywhere, even in the nursery. The warning that a kingdom could be lost for the want of a nail can be traced back to the 14th century; the following version of the familiar nursery rhyme was published in *Poor Richard's Almanack* in 1758 by Benjamin Franklin:

> For want of a nail the shoe was lost,
> For want of a shoe the horse was lost,
> and for want of a horse the rider was lost,
> being overtaken and slain by the enemy,
> all for the want of a horse-shoe nail.

We do not seek to explain the seed of instability with chaos, but rather to describe the growth of uncertainty *after* the initial seed is sown. In this case, explaining how it came to be that the rider was lost due to a missing nail, not the fact that the nail had gone missing. In fact, of course, there either was a nail or there was not. But Poor Richard tells us that if the nail hadn't been lost, then the kingdom wouldn't have been lost either. We will often explore the properties of chaotic systems by considering the impact of slightly different situations.

The study of chaos is common in applied sciences like astronomy, meteorology, population biology, and economics. Sciences making accurate observations of the world along with quantitative predictions have provided the main players in the development of chaos since the time of Isaac Newton. According to Newton's Laws, the future of the solar system is completely determined by its current state. The 19th-century scientist Pierre Laplace elevated this determinism to a key place in science. A world is deterministic if its current state completely defines its future. In 1820, Laplace conjured up an entity now known as 'Laplace's demon'; in doing so, he linked determinism and the ability to predict in principle to the very notion of success in science.

> We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

Note that Laplace had the foresight to give his demon three properties: exact knowledge of the Laws of Nature ('all the forces'), the ability to take a snapshot of the exact state of the universe ('all the positions'), and infinite computational resources ('an intellect vast enough to submit these data to analysis'). For Laplace's demon, chaos poses no barrier to prediction. Throughout this Introduction, we will consider the impact of removing one or more of these gifts.

From the time of Newton until the close of the 19th century, most scientists were also meteorologists. Chaos and meteorology are closely linked by the meteorologists' interest in the role uncertainty plays in weather forecasts. Benjamin Franklin's interest in meteorology extended far beyond his famous experiment of flying a kite in a thunderstorm. He is credited with noting the general movement of the weather from west towards the east and testing this theory by writing letters from Philadelphia to cities further east. Although the letters took longer to arrive than the weather, these are arguably early weather forecasts. Laplace himself discovered the law describing the decrease of atmospheric pressure with height. He also made fundamental contributions to the theory of errors: when we make an observation, the measurement is never exact in a mathematical sense, so there is always some uncertainty as to the 'True' value. Scientists often say that any uncertainty in an observation is due to ***noise***, without really defining exactly what the noise is, other than that which obscures our vision of whatever we are trying to measure, be it the length of a table, the number of rabbits in a garden, or the midday temperature. Noise gives rise to ***observational uncertainty***, chaos helps us to understand how small uncertainties can become large uncertainties, once we have a model for the noise. Some of the insights gleaned from chaos lie in clarifying the role(s) noise plays in the dynamics of uncertainty in the quantitative sciences. Noise has become much more interesting, as the study of chaos forces us to look again at what we might mean by the concept of a 'True' value.

Twenty years after Laplace's book on probability theory appeared, Edgar Allan Poe provided an early reference to what we would now call chaos in the atmosphere. He noted that merely moving our hands would affect the atmosphere all the way around the planet. Poe then went on to echo Laplace, stating that the mathematicians of the Earth could compute the progress of this hand-waving 'impulse', as it spread out and forever altered the state of the atmosphere. Of course, it is up to us whether or not we choose to wave our hands: free will offers another source of seeds that chaos might nurture.

In 1831, between the publication of Laplace's science and Poe's fiction, Captain Robert Fitzroy took the young Charles Darwin on his voyage of discovery. The observations made on this voyage led Darwin to his theory of natural selection. Evolution and chaos have more in common than one might think. First, when it comes to language, both 'evolution' and 'chaos' are used simultaneously to refer both to phenomena to be explained and to the theories that are supposed to do the explaining. This often leads to confusion between the description and the object described (as in 'confusing the map with the territory'). Throughout this Introduction we will see that confusing our mathematical models with the reality they aim to describe muddles the discussion of both. Second, looking more deeply, it may be that some ecosystems evolve as if they were chaotic systems, as it may well be the case that small differences in the environment have immense impacts. And evolution has contributed to the discussion of chaos as well. This chapter's opening quote comes from Ray Bradbury's 'A Sound Like Thunder', in which time-travelling big game hunters accidentally kill a butterfly, and find the future a different place when they return to it. The characters in the story imagine the impact of killing a mouse, its death cascading through generations of lost mice, foxes, and lions, and:

> all manner of insects, vultures, infinite billions of life forms are thrown into chaos and destruction . . . Step on a mouse and you leave your print, like a Grand Canyon, across Eternity. Queen Elizabeth might never be born, Washington might not cross the Delaware, there might never be a United States at all. So be careful. Stay on the Path. Never step off!

Needless to say, someone does step off the Path, crushing to death a beautiful little green and black butterfly. We can only consider these 'what if' experiments within the fictions of mathematics or literature, since we have access to only one realization of reality.

The origins of the term 'butterfly effect' are appropriately shrouded