

A  
BEAUTIFUL  
QUESTION



FINDING NATURE'S DEEP DESIGN

FRANK  
WILCZEK

WINNER OF THE NOBEL PRIZE IN PHYSICS

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# ADVANCE PRAISE FOR

## *A Beautiful Question*

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“A beautiful treatise on a beautiful universe, this delightful series of meditations on the nature of beauty and the physical universe roams from music to color vision to fundamental ideas at the very forefront of physics today. In lesser hands such a romp could easily degenerate into a kind of new-age mystical mumbo jumbo. However, Frank Wilczek is one of the deepest, most creative, and most knowledgeable theoretical physicists alive today. Read him or listen to him and you will never think about the universe the same way again. And if your experience is like mine over the years, you will definitely be the better for it.”

—LAWRENCE KRAUSS, author of *A Universe from Nothing* and *The Physics of Star Tr*

“A truly beautiful book, in design, in content, in the insights that Frank Wilczek shares. This book helps me see how one of the world’s leading thinkers thinks, using beauty as a tool, as a guide in finding not only the right problems but the right solutions. In Wilczek’s mind, there is no clear separation among physics, art, poetry, and music. Why do physicists call their theories beautiful? Immerse yourself in this book, wallow in it, sit back and relax as you wander through it, and you’ll soon understand.”

—RICHARD MULLER, author of *Physics for Future Presidents*

“If you’ve ever wondered what physicists mean when they describe a theory as ‘beautiful,’ *A Beautiful Question* is the ideal place to find out. Wilczek is both one of the greats of the subject and not afraid to engage nontechnically with the wonderful complexities and intangibilities of the mysterious beauty that lies at the core of our understanding of the physical world.”

—PETER WOIT, author of *Not Even Wrong*

“Frank Wilczek starts this fascinating book with the intriguing question *Does the world embody beautiful ideas?* What follows is a masterful intellectual journey, surveying a breathtaking tapestry of physics, art, and philosophy. One could ask Wilczek’s question differently: *Does this book embody beautiful ideas?* The answer would be a resounding Yes!”

—MARIO LIVIO, astrophysicist; author of *Brilliant Blunders*

“Having made fundamental contributions to our understanding of quantum field theory, Frank Wilczek now defines new frontiers with his creations in explanation: Maxwellian spider physics, the projective geometry of Renaissance art as a model of symmetry and invariance, links between anamorphic art and local symmetry, and more. Wilczek’s latest is a book with some beautiful answers.”

—FRANK CLOSE, author of *The Infinity Puzzle* and *Half-L*

“In contemporary art, beauty has faded and become a prosaic artifice, a distraction from deeper raw truths, maybe even ugly truths. To the exceptional physicist Frank Wilczek, beauty has proved a luminous ally, a faithful adviser in his discoveries of remarkable truths about the world. Ever in pursuit of truth, Wilczek guides us in a calm and winsome meditation on this subtle question: *Is the world beautiful?*”

—JANNA LEVIN, author of *How the Universe Got Its Sp*

“Anyone who has studied physics knows the startling beauty of those rare times when the clouds part and you see that math and reality are the same thing. With Wilczek’s new book, readers can catch a glimpse of that beauty without having to know the math.”

—NOAH SMITH, Stony Brook University; author of the blog “Noahpinio

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ALSO BY FRANK WILCZEK

*The Lightness of Being*  
*Fantastic Realities*  
*Longing for the Harmonies*



*This work was prepared especially for A Beautiful Question by He Shuifa, a modern master of traditional Chinese art and calligraphy. He is renowned for the vigor and subtlety of his brushwork and for the spiritual depth of his depictions of flowers, birds, and nature. A simple translation of the inscription is this: "Taiji double fish is the essence of Chinese culture. This image was painted by He Shuifa on a lake in early winter." The playful "double fish" aspect of Taiji comes to life in He Shuifa's image. The yin and yang resemble two carp playing together, and there are hints of their eyes and fins. In Henan, on the Yellow River, there is a waterfall called Dragon's Gate. Yulong carp attempt to jump the cataract, although it is very difficult for them. Those that succeed transform into lucky dragons. With a sense of humor, we may associate this event with the transformation of virtual into real particles, an essential quantum process that is now thought to underlie the origin of structure in the Universe (see [plates XX](#) and [AAA](#)). Alternatively we may identify ourselves with the carp, and their strivings with our quest for understanding.*

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*Finding Nature's Deep Design*

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TO MY FAMILY AND FRIENDS: BEAUTIFUL ANSWERS OF THE SECOND KIND

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## USER'S MANUAL

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- The “Timelines” are mainly focused on events mentioned or alluded to in the book. They do what timelines do. They are not intended to be complete histories of anything, and they aren’t.
- The “Terms of Art” section contains explanatory definitions and discussions of key terms and concepts that occur in the main text. As you can infer from its length, it is rather more than a standard glossary. It contains alternative perspectives on many ideas in the text, and develops a few in new directions.
- The “Notes” section contains material that might, in an academic setting, have gone into footnotes. It both qualifies the text and provides some more technical references on particular points. You’ll also find a pair of poems in there.
- The brief “Recommended Reading” section is not a routine list of popularizations, nor of textbooks, but a carefully considered set of recommendations for further exploration in the spirit of the text, emphasizing primary sources.

I hope you’ve already enjoyed the cover art and the frontispiece, which set the tone for our meditation beautifully.

There’s also a “User’s Manual”—but you knew that.

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# THE QUESTION

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This book is a long meditation on a single question:

Does the world embody beautiful ideas?

Our Question may seem like a strange thing to ask. Ideas are one thing, physical bodies are quite another. What does it mean to “embody” an “idea”?

Embodying ideas is what artists do. Starting from visionary conceptions, artists produce physical objects (or quasi-physical products, like musical scores that unfold into sound). Our Beautiful Question, then, is close to this one:

Is the world a work of art?

Posed this way, our Question leads us to others. If it makes sense to consider the world as a work of art, is it a successful work of art? Is the physical world, considered as a work of art, beautiful? For knowledge of the physical world we call on the work of scientists, but to do justice to our questions we must also bring in the insights and contributions of sympathetic artists.

## SPIRITUAL COSMOLOGY

Our Question is a most natural one, in the context of spiritual cosmology. If an energetic and powerful Creator made the world, it could be that what moved Him—or Her, or Them, or It—to create was precisely an impulse to make something beautiful. Natural though it may be, this is assuredly *not* an orthodox idea, according to most religious traditions. Many motivations have been ascribed to the Creator, but artistic ambition is rarely prominent among them.

In Abrahamic religions, conventional doctrine holds that the Creator set out to embody some combination of goodness and righteousness, and to create a monument to His glory. Animistic and polytheistic religions have envisaged beings and gods who create and govern different parts of the world with many kinds of motives, running the gamut from benevolence to lust to carefree exuberance.

On a higher theological plane, the Creator’s motivations are sometimes said to be so awesome that

finite human intellects can't hope to comprehend them. Instead we are given partial revelations, which are to be believed, not analyzed. Or, alternatively, God is Love. None of those contradictory orthodoxies offers compelling reasons to expect that the world embodies beautiful ideas; nor do they suggest that we should strive to find such ideas. Beauty can form part of their cosmic story, but it is generally regarded as a side issue, not the heart of the matter.

Yet many creative spirits have found inspiration in the idea that the Creator might be, among other things, an artist whose esthetic motivations we can appreciate and share—or even, in daring speculation, that the Creator is *primarily* a creative artist. Such spirits have engaged our Question, in varied and evolving forms, across many centuries. Thus inspired, they have produced deep philosophy, great science, compelling literature, and striking imagery. Some have produced works that combine several, or all, of those features. These works are a vein of gold running back through our civilization.

Galileo Galilei made the beauty of the physical world central to his own deep faith, and recommended it to all:

The greatness and the glory of God shine forth marvelously in all His works, and is to be read above all in the open book of the heavens.

. . . as did Johannes Kepler, Isaac Newton, and James Clerk Maxwell. For all these searchers, finding beauty embodied in the physical world, reflecting God's glory, was the goal of their search. It inspired their work, and sanctified their curiosity. And with their discoveries, their faith was rewarded.

While our Question finds support in spiritual cosmology, it can also stand on its own. And though its positive answer may inspire a spiritual interpretation, it does not require one.

We will return to these thoughts toward the end of our meditation, by which point we will be much better prepared to appraise them. Between now and then, the world can speak for itself.

## HEROIC VENTURES

Just as art has a history, with developing standards, so does the concept of the world as a work of art. In art history, we are accustomed to the idea that old styles are not simply obsolete, but can continue to be enjoyed on their own terms, and also offer important context for later developments. Though that idea is much less familiar in science, and in science it is subject to important limitations, the historical approach to our Question offers many advantages. It allows us—indeed, forces us—to proceed from simpler to more complex ideas. At the same time, by exploring how great thinkers struggled and often went astray, we gain perspective on the initial strangeness of ideas that have become, through familiarity, too “obvious” and comfortable. Last but by no means least, we humans are especially adapted to think in story and narrative, to associate ideas with names and faces, and to find tales of conflicts and their resolution compelling, even when they are conflicts of ideas, and no blood gets spilled. (Actually, a little does . . .)

For these reasons we will sing, to begin, songs of heroes: Pythagoras, Plato, Filippo Brunelleschi, Newton, Maxwell. (Later a major heroine, Emmy Noether, will enter too.) Real people went by those names—very interesting ones! But for us they are not merely people, but also legends and symbols. I've portrayed them, as I think of them, in that style, emphasizing clarity and simplicity over scholarly nuance. Here biography is a means, not an end. Each hero advances our meditation several steps:

- 
- *Pythagoras* discovered, in his famous theorem about right-angled triangles, a most fundamental relationship between numbers, on the one hand, and sizes and shapes, on the other. Because Number is the purest product of Mind, while Size is a primary characteristic of Matter, that discovery revealed a hidden unity between Mind and Matter.

Pythagoras also discovered, in the laws of stringed instruments, simple and surprising relationships between numbers and musical harmony. That discovery completes a trinity, Mind-Matter-Beauty, with Number as the linking thread. Heady stuff! It led Pythagoras to surmise that All Things Are Number. With these discoveries and speculations, our Question comes to life.

- *Plato* thought big. He proposed a geometric theory of atoms and the Universe, based on five symmetrical shapes, which we now call the Platonic solids. In this audacious model of physical reality, Plato valued beauty over accuracy. The details of his theory are hopelessly wrong. Yet it provided such a dazzling vision of what a positive answer to our Question might look like that it inspired Euclid, Kepler, and many others to brilliant work centuries later. Indeed, our modern, astoundingly successful theories of elementary particles, codified in our Core Theory (see page 8), are rooted in heightened ideas of symmetry that would surely make Plato smile. And when trying to guess what will come next, I often follow Plato's strategy, proposing objects of mathematical beauty as models for Nature.

Plato was also a great literary artist. His metaphor of the Cave captures important emotional and philosophical aspects of our relationship, as human inquirers, with reality. At its core is the belief that everyday life offers us a mere shadow of reality, but that through adventures of mind, and sensory expansion, we can get to its essence—and that the essence is clearer and more beautiful than its shadow. He imagined a mediating *demiurge*, which can be translated as *Artisan*, who rendered the realm of perfect, eternal Ideas into its imperfect copy, the world we experience. Here the concept of the world as a work of art is explicit.

- *Brunelleschi* brought new ideas to geometry from the needs of art and engineering. His *projective geometry*, in dealing with the actual appearance of things, brought in ideas—relativity, invariance, symmetry—not only beautiful in themselves, but pregnant with potential.
- *Newton* brought the mathematical understanding of Nature to entirely new levels of ambition and precision.

A common theme pervades Newton's titanic work on light, the mathematics of calculus, motion, and mechanics. It is the method he called Analysis and Synthesis. The method of Analysis and Synthesis suggests a two-stage strategy to achieve understanding. In the analysis stage, we consider the smallest parts of what we are studying their "atoms," using the word figuratively. In a successful analysis, we identify small parts that have simple properties that we can summarize in precise laws. For example:

- In the study of light, the atoms are beams of pure spectral colors.
- In the study of calculus, the atoms are infinitesimals and their ratios.
- In the study of motion, the atoms are velocity and acceleration.
- In the study of mechanics, the atoms are forces.

(We'll discuss these in more depth later.) In the synthesis stage we build up, by logical and mathematical reasoning, from the behavior of individual atoms to the description of systems that contain many atoms.

When thus stated broadly, Analysis and Synthesis may not seem terribly impressive. It is, after all, closely related to common rules of thumb, e.g., “to solve a complex problem, divide and conquer”—hardly an electrifying revelation. But Newton demanded precision and completeness of understanding, saying,

'Tis much better to do a little with certainty & leave the rest for others that come after than to explain all things by conjecture without making sure of any thing.

And in these impressive examples, he achieved his ambitions. Newton showed, convincingly, that Nature herself proceeds by Analysis and Synthesis. There really is simplicity in the “atoms,” and Nature really does operate by letting them do their thing.

Newton also, in his work on motion and mechanics, enriched our concept of what physical laws are. His laws of motion and of gravity are *dynamical* laws. In other words, they are laws of change. Laws of this kind embody a different concept of beauty than the static perfection beloved of Pythagoras and (especially) Plato.

Dynamical beauty transcends specific objects and phenomena, and invites us to imagine the expanse of possibilities. For example, the sizes and shapes of actual planetary orbits are not simple. They are neither the (compounded) circles of Aristotle, Ptolemy, and Nicolaus Copernicus, nor even the more nearly accurate ellipses of Kepler, but rather curves that must be calculated numerically, as functions of time, evolving in complicated ways that depend on the positions and masses of the Sun and the other planets. There is great beauty and simplicity here, but it is only fully evident when we understand the deep design. The appearance of particular objects does not exhaust the beauty of the laws.

- *Maxwell* was the first truly modern physicist. His work on electromagnetism ushered in both a new concept of reality and a new method in physics. The new concept, which Maxwell developed from the intuitions of Michael Faraday, is that the primary ingredients of physical reality are not point-like *particles*, but rather space-filling *fields*. The new method is *inspired guesswork*. In 1864 Maxwell codified the known laws of electricity and magnetism into a system of equations, but discovered the resulting system was inconsistent. Like Plato, who shoehorned five perfect solids into four elements plus the Universe, Maxwell did not give up. He saw that by adding a new term he could both make the equations appear more symmetric and make them mathematically consistent. The resulting system, known as the Maxwell equations, not only unified electricity and magnetism, but derived light as a consequence, and survives to this day as the secure foundation of those subjects.

By what is the physicist's “inspired guesswork” inspired? Logical consistency is necessary, but hardly sufficient. Rather it was beauty and symmetry that guided Maxwell and his followers—that is, all modern physicists—closer to truth, as we shall see.

Maxwell also, in his work on color perception, discovered that Plato's metaphorical Cave reflects something quite real and specific: the paltriness of our sensory experience,

relative to available reality. And his work, by clarifying the limits of perception, allows us to transcend those limits. For the ultimate sense-enhancing device is a searching mind.

## QUANTUM FULFILLMENT

The definitive answer “yes” to our Question came only in the twentieth century, with the development of quantum theory.

The quantum revolution gave this revelation: we’ve finally learned what Matter is. The necessary equations are part of the theoretical structure often called the Standard Model. That yawn-inducing name hardly does the achievement justice, and I’m going to continue my campaign, begun in *The Lightness of Being*, to replace it with something more appropriately awesome:

### Standard Model → Core Theory

This change is more than justified, because

1. “Model” connotes a disposable makeshift, awaiting replacement by the “real thing.” But the Core Theory is already an accurate representation of physical reality, which any future, hypothetical “real thing” must take into account.
2. “Standard” connotes “conventional,” and hints at superior wisdom. But no such superior wisdom is available. In fact, I think—and mountains of evidence attest—that while the Core Theory will be supplemented, its core will persist.

The Core Theory embodies beautiful ideas. The equations for atoms and light are, almost literally the same equations that govern musical instruments and sound. A handful of elegant designs support Nature’s exuberant construction, from simple building blocks, of the material world.

Our Core Theories of the four forces of Nature—gravity, electromagnetism, and the strong and weak forces—embody, at their heart, a common principle: *local symmetry*. As you will read, this principle both fulfills and transcends the yearnings of Pythagoras and Plato for harmony and conceptual purity. As you will see, this principle both builds upon and transcends the artistic geometry of Brunelleschi and the brilliant insights of Newton and Maxwell into the nature of color.

The Core Theory completes, for practical purposes, the analysis of matter. Using it, we can deduce what sorts of atomic nuclei, atoms, molecules—and stars—exist. And we can reliably orchestrate the behavior of larger assemblies of these elements, to make transistors, lasers, or Large Hadron Colliders. The equations of the Core Theory have been tested with far greater accuracy, and under far more extreme conditions, than are required for applications in chemistry, biology, engineering, or astrophysics. While there certainly are many things we don’t understand—I’ll mention some important ones momentarily!—we do understand the Matter we’re made from and that we encounter in normal life (even if we’re chemists, engineers, or astrophysicists).

Despite its overwhelming virtues, the Core Theory is imperfect. Indeed, precisely because it is such a faithful description of reality, we must, in pursuit of our Question, hold it to the highest esthetic standards. So scrutinized, the Core Theory reveals flaws. Its equations are lopsided, and they

contain several loosely connected pieces. Furthermore, the Core Theory does not account for so-called dark matter and dark energy. Although those tenuous forms of matter are negligible in our immediate neighborhood, they persist in the interstellar and intergalactic voids, and thereby come to dominate the overall mass of the Universe. For those and other reasons, we cannot remain satisfied.

Having tasted beauty at the heart of the world, we hunger for more. In this quest there is, I think, a more promising guide than beauty itself. I shall show you some hints that suggest concrete possibilities for improving our description of Nature. As I aspire to inspired guesswork, beauty is my inspiration. Several times it's worked well for me, as you'll see.

## VARIETIES OF BEAUTY

Different artists have different styles. We don't expect to find Renoir's shimmering color in Rembrandt's mystic shadows, or the elegance of Raphael in either. Mozart's music comes from a different world entirely, the Beatles' from another, and Louis Armstrong's from yet another. Likewise, the beauty embodied in the physical world is a particular kind of beauty. Nature, as an artist, has a distinctive style.

To appreciate Nature's art, we must enter her style with sympathy. Galileo, ever eloquent, expressed it this way:

Philosophy [Nature] is written in that great book which ever is before our eyes—I mean the universe—but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.

Today we've penetrated much further into the great book, and discovered that its later chapters use a more imaginative, less familiar language than the Euclidean geometry Galileo knew. To become a fluent speaker in it is the work of a lifetime (or at least of several years in graduate school). But just a graduate degree in art history is not a prerequisite for engaging with the world's best art and finding that a deeply rewarding experience, so I hope, in this book, to help you engage with Nature's art, by making her style accessible. Your effort will be rewarded, for as Einstein might have said,

Subtle is the Lord, but malicious She is not.

Two obsessions are the hallmarks of Nature's artistic style:

- Symmetry—a love of harmony, balance, and proportion
- Economy—satisfaction in producing an abundance of effects from very limited means

Watch for these themes as they recur, grow, and develop throughout our narrative and give it unity. Our appreciation of them has evolved from intuition and wishful thinking into precise, powerful, and

fruitful methods.

Now, a disclaimer. Many varieties of beauty are underrepresented in Nature's style, as expressed in her fundamental operating system. Our delight in the human body and our interest in expressive portraits, our love of animals and of natural landscapes, and many other sources of artistic beauty are not brought into play. Science isn't everything, thank goodness.

## CONCEPTS AND REALITIES; MIND AND MATTER

Our Question can be read in two directions. Most obviously, it is a question about the world. That is the direction we've emphasized so far. But the other direction is likewise fascinating. When we find that *our* sense of beauty is realized in the physical world, we are discovering something about the world, but also something about ourselves.

Human appreciation of the fundamental laws of Nature is a recent development on evolutionary or even historical time scales. Moreover, those laws reveal themselves only after elaborate operations—looking through sophisticated microscopes and telescopes, tearing atoms and nuclei apart, and processing long chains of mathematical reasoning—that do not come naturally. Our sense of beauty is not in any very direct way adapted to Nature's fundamental workings. Yet just as surely, our sense of beauty is excited by what we find there.

What explains that miraculous harmony of Mind and Matter? Without an explanation of that miracle, our Question remains mysterious. It is an issue our meditation will touch upon repeatedly. For now, two brief anticipations:

1. We human beings are, above all, visual creatures. Our sense of vision, of course, and in a host of less obvious ways our deepest modes of thought, are conditioned by our interaction with light. Each of us, for example, is born to become an accomplished, if unconscious, practitioner of projective geometry. That ability is hardwired into our brain. It is what allows us to interpret the two-dimensional image that arrives on our retinas as representing a world of objects in three-dimensional space.

Our brains contain specialized modules that allow us to construct, very quickly and without conscious effort, a dynamic worldview based on three-dimensional objects located in three-dimensional space. We do this beginning from two two-dimensional images on the retinas of our eyes (which, in turn, are the product of light rays emitted or reflected from the surfaces of external objects, which propagate to us in straight lines). To work back from the images we receive to the objects that cause them is a tricky problem in inverse projective geometry. In fact, as stated, it is an impossible problem, because there's not nearly enough information in the projections to do an unambiguous reconstruction. A basic problem is that even to get started we need to separate objects from their background (or foreground). We exploit all kinds of tricks based on typical properties of objects we encounter, such as their color or texture contrast and distinctive boundaries, to do that job. But even after that step is accomplished, we are left with a difficult geometrical problem, for which Nature has helpfully provided us, in our visual cortex, an excellent specialized processor.

Another important feature of vision is that light arrives to us from very far away, and gives us a window into astronomy. The regular apparent motion of stars and the slightly



less regular apparent motion of planets gave early hints of a lawful Universe, and provided an early inspiration and testing ground for the mathematical description of Nature. Like a good textbook, it contains problems with varying degrees of difficulty.

In the more advanced, modern parts of physics we learn that light itself is a form of matter, and indeed that matter in general, when understood deeply, is remarkably light-like. So again, our interest in and experience with light, which is deeply rooted in our essential nature, proves fortunate.

Creatures that, like most mammals, perceive the world primarily through the sense of smell would have a much harder time getting to physics as we know it, even if they were highly intelligent in other ways. One can imagine dogs, say, evolving into extremely intelligent social creatures, developing language, and experiencing rich lives full of interest and joy, but devoid of the specific kinds of curiosity and outlook, based on visual experience, that lead to our kind of deep understanding of the physical world. Their world would be rich in reactions and decays—they'd have great chemistry sets, elaborate cuisines, aphrodisiacs, and, à la Proust, echoing memories. Projective geometry and astronomy, maybe not so much. We understand that smell is a chemical sense, and we are beginning to understand its foundation in molecular events. But the “inverse” problem of working from smell back to molecules and their laws, and eventually to physics as we know it, seems to me hopelessly difficult.

Birds, on the other hand, are visual creatures, like us. Beyond that, their way of life would give them an extra advantage over humans, in getting started on physics. For birds, with their freedom of flight, experience the essential symmetry of three-dimensional space in an intimate way that we do not. They also experience the basic regularities of motion, and especially the role of inertia, in their everyday lives, as they operate in a nearly frictionless environment. Birds are born, one might say, with intuitive knowledge of classical mechanics and Galilean relativity, as well as of geometry. If some species of bird evolved high abstract intelligence—that is, if they ceased being birdbrains—their physics would develop rapidly. Humans, on the other hand, have to unlearn the friction-laden Aristotelean mechanics they use in everyday life, in order to achieve deeper understanding. Historically that involved quite a struggle!

Dolphins, in their watery environment, and bats, with their echolocation, give us other interesting variations on these themes. But I will not develop those here.

A general philosophical point, which these considerations illustrate, is that the world does not provide its own unique interpretation. The world offers many possibilities for different sensory universes, which support very different interpretations of the world's significance. In this way our so-called Universe is already very much a multiverse.

2. Successful perception involves sophisticated inference, because the information we sample about the world is both very partial and very noisy. For all our innate powers, we must also learn how to see by interacting with the world, forming expectations, and comparing our predictions with reality. When we form expectations that turn out to be correct, we experience pleasure and satisfaction. Those reward mechanisms encourage successful learning. They also stimulate—indeed, at base they *are*—our sense of beauty.

Putting those observations together, we discover an explanation of why we find interesting phenomena (phenomena we can learn from!) in physics beautiful. An important consequence is that we especially value experience that is surprising, but not

too surprising. Routine, superficial recognition will not challenge us, and may not be rewarded as active learning. On the other hand, patterns whose meaning we cannot make sense of at all will not offer rewarding experience either; they are noise.

And here we are lucky too, in that Nature employs, in her basic workings, symmetry and economy of means. For these principles, like our intuitive understanding of light, promote successful prediction and learning. From the appearance of part of a symmetric object we can predict (successfully!) the appearance of the rest; from the behavior of parts of natural objects we can predict (sometimes successfully!) the behavior of wholes. Symmetry and economy of means, therefore, are exactly the sorts of things we are apt to experience as beautiful.

## NEW IDEAS AND INTERPRETATIONS

Together with new appreciations of some very old and some less old ideas, you will find in this book several essentially new ones. Here I'd like to mention some of the most important.

My presentation of the Core Theory as geometry, and my speculations about the next steps beyond it, are adaptations of my technical work in fundamental physics. That work builds, of course, on the work of many others. My use of color fields as an example of extra dimensions, and my exploitation of the possibilities they open up for illustrating local symmetry, are (as far as I know) new.

My theory that promotion of learning underlies, and is the evolutionary cause of, our sense of beauty in important cases, and the application of that theory to musical harmony, which offers a rational explanation for Pythagoras's discoveries in music, form a constellation of ideas I've entertained privately for a long time but present here for the first time publicly. Caveat emptor.

My discussion of the expansion of color perception draws on an ongoing program of practical research that I hope will lead to commercial products. Patents have been applied for.

I'd like to think that Niels Bohr would approve of my broad interpretation of complementarity, and might even acknowledge his paternity—but I'm not sure he would.

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# PYTHAGORAS I: THOUGHT AND OBJECT

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## THE SHADOW PYTHAGORAS

There was a person named Pythagoras who lived and died around 570–495 BCE, but very little is known about him. Or rather a lot is “known” about him, but most of it is surely wrong, because the documentary trail is littered with contradictions. It combines the sublime, the ridiculous, the unbelievable, and the just plain weird.

Pythagoras was said to be the son of Apollo, to have a golden thigh, and to glow. He may or may not have advocated vegetarianism. Among his most notorious sayings is an injunction not to eat beans because “beans have a soul.” Yet several early sources explicitly deny that Pythagoras said or believed anything of the sort. More reliably, Pythagoras believed in, and taught, the transmigration of souls. There are several stories—each, to be sure, dubious—that corroborate this. According to Aulus Gellius, Pythagoras remembered four of his own past lives, including one as a beautiful courtesan named Alco. Xenophanes recounts that Pythagoras, upon hearing the cries of a dog who was being beaten, rushed to halt the beating, claiming to recognize the voice of a departed friend. Pythagoras also, like Saint Francis centuries later, preached to animals.

The *Stanford Encyclopedia of Philosophy*—a free and extremely valuable online resource, by the way—sums it up as follows:

The popular modern image of Pythagoras is that of a master mathematician and scientist. The early evidence shows, however, that, while Pythagoras was famous in his own day and even 150 years later in the time of Plato and Aristotle, it was not mathematics or science upon which his fame rested. Pythagoras was famous

1. As an expert on the fate of the soul after death, who thought that the soul was immortal and went through a series of reincarnations
2. As an expert on religious ritual
3. As a wonder-worker who had a thigh of gold and who could be two places at the same time
4. As the founder of a strict way of life that emphasized dietary restrictions, religious ritual and rigorous self discipline

A few things do seem clear. The historical Pythagoras was born on the Greek island of Samos,

traveled widely, and became the inspiration for and founder of an unusual religious movement. His cult flourished briefly in Crotona, in southern Italy, and developed chapters in several other places before being everywhere suppressed. The Pythagoreans formed secretive societies, on which the initiates' lives centered. These communities, which included both men and women, promoted a kind of intellectual mysticism that seemed marvelous, yet strange and threatening, to most of their contemporaries. Their worldview centered on worshipful admiration of numbers and musical harmony, which they saw as reflecting the deep structure of reality. (As we'll see, they were on to something.)

## THE REAL PYTHAGORAS

Here again is the *Stanford Encyclopedia*:

The picture of Pythagoras that emerges from the evidence is thus not of a mathematician, who offered rigorous proofs, or of a scientist, who carried out experiments to discover the nature of the natural world, but rather of someone who sees special significance in and assigns special prominence to mathematical relationships that were in general circulation.

Bertrand Russell was pithier:

A combination of Einstein and Mary Baker Eddy.

To scholars of factual biography, it is a major problem that later followers of Pythagoras ascribed their own ideas and discoveries to Pythagoras himself. In that way they hoped both to give their ideas authority and, by enhancing Pythagoras's reputation, to promote their community—the community he founded. Thus magnificent discoveries in different fields of mathematics, physics, and music, as well as an inspiring mysticism, a seminal philosophy, and a pure morality were all portrayed as the legacy of a single godlike figure. That awesome figure is, for us, the *real* Pythagoras.

It is not altogether inappropriate to assign the (historical) shadow Pythagoras credit for the real Pythagoras, because the latter's great achievements in mathematics and science emerged from the work of life the former inspired, and the community he founded.

(Those so inclined might draw parallels to the differing careers in life, and afterward, of other major religious figures . . .)

Thanks to Raphael, we know what the real Pythagoras looked like. In [plate B\\*](#) he is captured deep in concentration as he writes in a great book, surrounded by admirers.

## ALL THINGS ARE NUMBER

It is difficult to make out what Pythagoras is writing, but I like to pretend it is some version of his most fundamental credo:

It is also difficult to know, at this separation in time and space, exactly what Pythagoras meant by that. So we get to use our imagination.

## PYTHAGORAS'S THEOREM

For one thing, Pythagoras was mightily impressed by Pythagoras's theorem. So much so that when he discovered it, in a notable lapse from vegetarianism, he offered a hecatomb—the ritual sacrifice of one hundred oxen, followed by feasting—to the Muses, in thanks.

Why the fuss?

Pythagoras's theorem is a statement about right triangles; that is, triangles that contain a 90-degree angle, or, in other words, a square corner. The theorem tells you that if you erect squares on the different sides of such a triangle, then the sum of the areas of the two smaller squares adds up to the area of the largest square. A classic example is the 3-4-5 right triangle, shown in [figure 1](#):

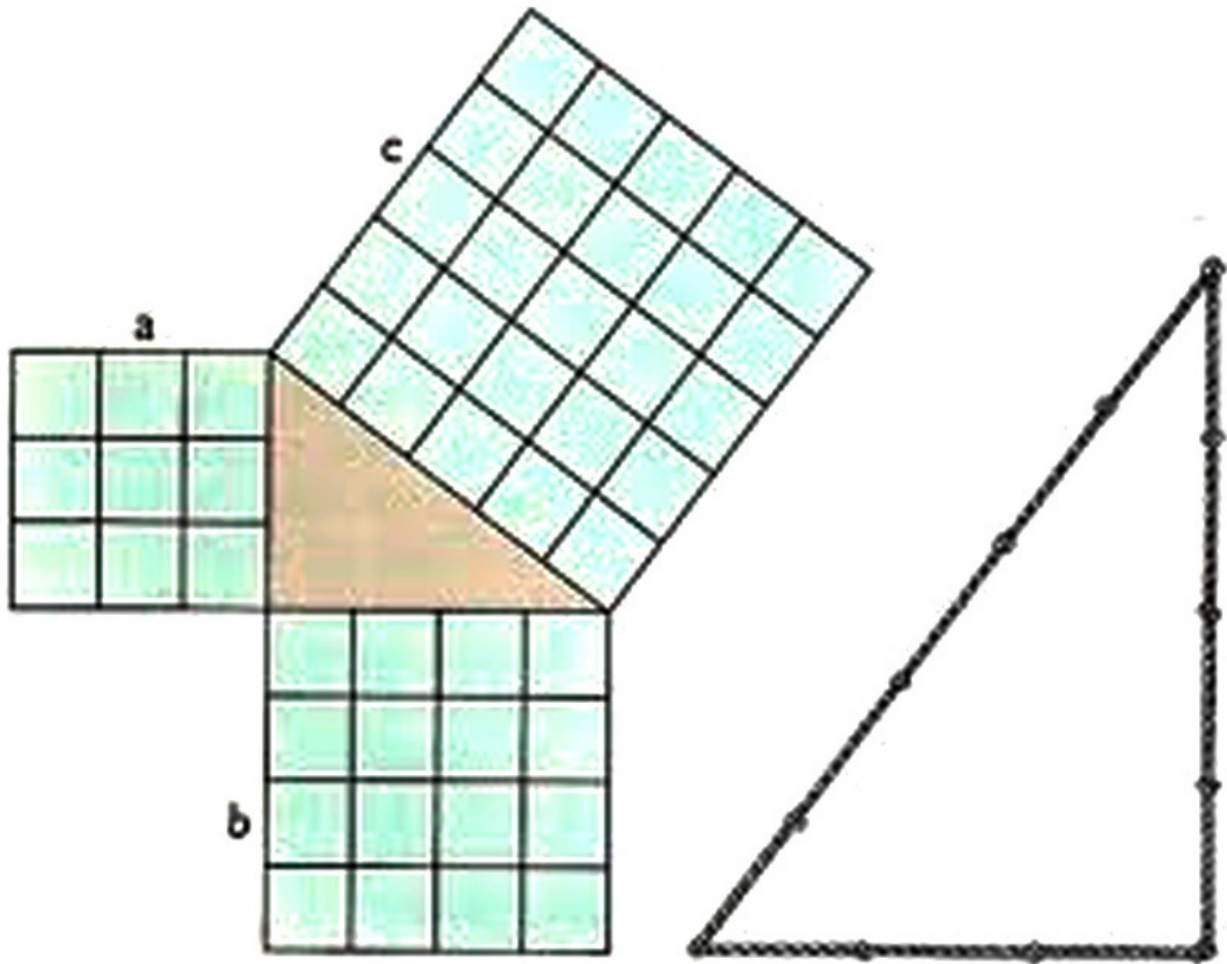


FIGURE 1. THE 3-4-5 RIGHT TRIANGLE, A SIMPLE CASE OF PYTHAGORAS'S THEOREM.

The areas of the two smaller squares are  $3^2 = 9$  and  $4^2 = 16$ , as we can see, in the spirit of Pythagoras by *counting* their subunits. The area of the largest square is  $5^2 = 25$ . And we verify  $9 + 16 = 25$ .

By now Pythagoras's theorem is familiar to most of us, if only as a dim memory from school geometry. But if you listen to its message afresh, with Pythagoras's ears, so to speak, you realize that it is saying something quite startling. It is telling you that the *geometry* of objects embodies hidden *numerical* relationships. It says, in other words, that Number describes, if not yet everything, at least something very important about physical reality, namely the sizes and shapes of the objects that inhabit it.

Later in this meditation we will be dealing with much more advanced and sophisticated concepts, and I'll have to resort to metaphors and analogies to convey their meaning. The special joy one finds in precise mathematical thinking, when sharply defined concepts fit together perfectly, is lost in translation. Here we have an opportunity to experience that special joy. Part of the magic of Pythagoras's theorem is that one can prove it with minimal preparation. The best proofs are unforgettable, and their memory lasts a lifetime. They've inspired Aldous Huxley and Albert Einstein—not to mention Pythagoras!—and I hope they'll inspire you.

### *Guido's Proof*

“So simple!”

That is what Guido, the young hero of Aldous Huxley's short story “Young Archimedes,” says, as he describes his demonstration of Pythagoras's theorem. Guido's proof is based on the shapes displayed in [plate C](#).

### *Guido's Plaything*

Let's spell out what was obvious to Guido at a glance.

Each of the two large tiled squares contains four colored triangles that are matched in the other large square. All the colored triangles are right triangles, and all are the same size. Let's say the length of the smallest side is  $a$ , the next smallest  $b$ , and the longest (the hypotenuse)  $c$ . Then it's easy to see that the sides of both large (total) squares have length  $a + b$ , and in particular that those two squares have equal areas. So the non-triangular parts of the large squares must also have equal areas.

But what are those equal areas? In the first large square, on the left, we have a blue square with side  $a$ , and a red square with side  $b$ . They have areas  $a^2$  and  $b^2$ , and their combined area is  $a^2 + b^2$ . In the second large square, on the right, we have a gray square with side  $c$ . Its area is  $c^2$ . Recalling the preceding paragraph, we conclude that

$$a^2 + b^2 = c^2$$

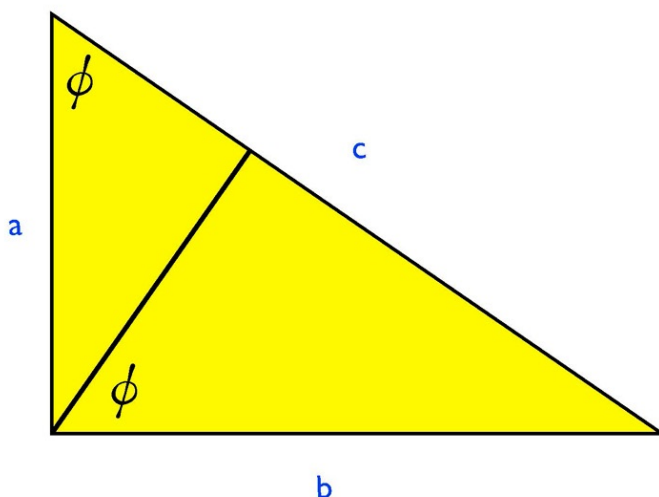
... which is Pythagoras's theorem!

### *Einstein's Proof(?)*

In Einstein's *Autobiographical Notes* he recalls,

I remember that an uncle told me about the Pythagorean theorem before the holy geometry booklet had come into my hands. After much effort I succeeded in “proving” this theorem on the basis of the similarity of triangles; in doing so it seemed to me “evident” that the relations of the sides of the right-angled triangles would have to be determined by one of the acute angles.

There is not really enough detail in that account to reconstruct Einstein’s demonstration with certainty, but here, in figure 2, is my best guess. That guess deserves to be right, because this is the simplest and most beautiful proof of Pythagoras’s theorem. In particular, this proof makes it brilliantly clear why the *squares* of the lengths are what’s involved in the theorem.



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FIGURE 2. A PLAUSIBLE RECONSTRUCTION OF EINSTEIN’S PROOF, FROM *AUTOBIOGRAPHICAL NOTES*.

### *A Polished Jewel*

We start from the observation that right triangles that include a common angle  $\phi$  are all similar to one another, in the precise sense that you can get from any one to any other by an overall rescaling (magnification or shrinking). Also: if we rescale the length of the triangle by some factor, then we will rescale the area by the square of that factor.

Now consider the three right triangles that appear in figure 2: the total figure, and the two sub-triangles it contains. Each of them contains the angle  $\phi$ , so they are similar. Their areas are therefore proportional to  $a^2$ ,  $b^2$ ,  $c^2$ , going from smallest to largest. But because the two sub-triangles add up to the total triangle, the corresponding areas must also add up, and therefore

$$a^2 + b^2 = c^2$$

... Pythagoras’s theorem pops right out!

### *A Beautiful Irony*

It is a beautiful irony that Pythagoras's theorem can be used to undermine his doctrine All Things Are Number.

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That scandalous result is the one discovery of the Pythagorean school that was not attributed to Pythagoras, but rather to his pupil Hippiasus. Shortly after his discovery, Hippiasus drowned at sea. Whether his death should be attributed to the wrath of the gods, or the wrath of the Pythagoreans, is a debated point.

Hippiasus's reasoning is very clever, but not overly complicated. Let's waltz through it.

We consider isosceles right triangles with two equal sides—in other words,  $a = b$ . Pythagoras's theorem tells us that

$$2 \times a^2 = c^2$$

Now let's suppose that the lengths  $a$  and  $c$  are both whole numbers. If *all* things are numbers, they'd better be! But we'll find that it's impossible.

If both  $a$  and  $c$  are even numbers, we can consider a similar triangle of half the size. We can keep halving until we reach a triangle where at least one of  $a$ ,  $c$  is odd.

But whichever choice we make, we quickly derive a contradiction.

First let's suppose that  $c$  is odd. Then so is  $c^2$ . But  $2 \times a^2$  is obviously even because it contains a factor 2. So we can't have  $2 \times a^2 = c^2$ , as Pythagoras's theorem tells us. Contradiction!

Alternatively, suppose that  $c$  is even, say  $c = 2 \times p$ . Then  $c^2 = 4 \times p^2$ . Then Pythagoras's theorem tells us, after we divide both sides by 2, that  $a^2 = 2 \times p^2$ . And so  $a$  can't be odd, by the same reasoning as before. Contradiction!

So all things can't be whole numbers, after all. There cannot be an atom of length, such that all possible lengths are whole number multiples of that atom's length.

It doesn't seem to have occurred to the Pythagoreans that one might draw a different conclusion, saving All Things Are Number. After all, one *can* imagine a world where space is constructed from many identical atoms. Indeed, my friends Ed Fredkin and Stephen Wolfram advocate models of our world based on cellular automata, which have exactly this property. And your computer screen, based on atoms of light we call pixels, shows that such a world can look pretty realistic! Logically, the correct conclusion to draw is that in such a world, one cannot construct exact isosceles right triangles. Something has to go slightly wrong. The "right" angle might fail to be exactly 90 degrees, or the two shorter sides might fail to be perfectly equal or—as on the computer screen—the sides of your triangles might fail to be exactly straight.

This is not the option Greek mathematicians chose. Rather, they considered geometry in its more appealing continuous form, where we allow exact right angles and exact equality of sides to coexist. (This is also the choice that has proved most fruitful for physics, as we'll learn from Newton.) To do this, they had to prioritize geometry over arithmetic, because—as we've seen—the whole numbers are inadequate to describe even very simple geometric figures. Thus they abandoned the letter, though not the spirit, of All Things Are Number.

## THOUGHT AND OBJECT

For the true essence of Pythagoras's credo is not a literal assertion that the world must embody



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